Equilibrium Points in Games with Virtual Strategies.

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ABSTRACT

Since the concept of virtual reality was introduced to game theory, there has been a need to analyze equilibrium points in virtual games. In order to do this, some virtual games are analyzed with a view to identifying their equilibrium points. From the analysis, some of the games have multiple equilibrium points, while some have single equilibrium point. Some have pure strategy equilibria, while some have mixed strategy equilibria. Above all, the concept of perfect and imperfect virtual equilibria is discovered.

(Keywords: game theory, games theory, virtual games, equilibrium points)

INTRODUCTION

Analysis of equilibria has been at the heart of game theory since John Nash introduced the concept in 1950 in his famous paper "Equilibrium Points in n-persons Game" (Nash, 1950). Nash equilibrium is based on the assumption that players are rational. According to Fudenberg and Tyrole (1991), the fact that Nash equilibria pass the test of being consistent predictions does not make them good predictions, and in situations it seems rash to think that a precise prediction is available. Hence, the likely outcome of a game depends on more information than is revealed in the strategic form. Here issues like the experience of players, morality, religious beliefs, cultural background, educational background, etc., all come into play, and of course, players are not always rational.

The concept of virtual games and virtual equilibria was introduced by Nwobi-Okoye (2009; 2010b). Virtual games use strategies based on virtual reality known as virtual strategies. Virtual reality occurs when the payoff determining factors assume certain conditions exist which in fact do not. Virtual reality strategies use deceptive perceptions or illusions to improve payoffs for the strategist (Nwobi-Okoye, 2009; Nwobi-Okoye, 2010a; Nwobi-Okoye, 2010b).

Virtual equilibria occur in numerous scenarios in life. The whole story of stability in the ecosystem could be traced to virtual equilibria. Nature uses a powerful virtual strategy simply called camouflage by naturalists to help maintain a delicate balance among species in the ecosystem. Such virtual strategies in nature include: deceptive colors (for instance green snakes in green grass, brown snakes in the desert, crowing snakes, chameleonic color changes etc) and deceptive noises (for instance crowing snakes). Hence, without virtual equilibria there would be chaos in nature.

In the military and warfare, camouflage, a powerful virtual strategy, is also generally used. The camouflages range from deceptive uniforms, equipment, noise, etc.

In the Christian holy book, the Bible, Jacob used virtual strategies extensively. First, he used a virtual strategy by covering himself with goat's skin in order to resemble his brother, Esau, who was very hairy. In so doing he deceived his blind father and collected his last blessing which was meant for his brother Esau (The Book of Genesis 27). Secondly, he used a virtual strategy to initiate a birth processes that favored his kind among his brother in-laws' sheep (The Book of Genesis 30, 27-43). He was therefore able to introduce artificial agents to obtain higher payoffs. The virtual strategy mentioned above that Jacob used to take their father's blessing from his brother, Esau, is perhaps the most famous virtual strategy in history, and gave rise to the nation of Israel.

In central Africa, some communities use pre-recorded sound of buzzing bees donated by a non-governmental organization (NGO) to scare away elephants that destroy their crops. Of course this is a powerful virtual strategy. The use of scarecrows to ward off pests is also a powerful virtual strategy.

Recently an art gallery owner, Shane Record from Kent, England, who entered for a BBC science enthusiast competition, narrowly lost being short listed. Shane wanted to test his observation that more people come into his gallery when he puts a mannequin by the artwork. The mannequin he said gives the illusion that they will not be alone in the gallery, thereby making visitors more likely to enter (BBC, 2010). His observation if proven would constitute a powerful virtual strategy.

This paper discusses equilibrium in the context of virtual games. Trembling and perfect equilibrium introduced here are remarkably different from their earlier concept as introduced by Selten (1965; 1975). Secondly, Bayesian, correlated and mixed strategy equilibria is discussed in the context of virtual games. Also discussed is the concept of multiple equilibria and focal points for virtual games with examples of how and where they could occur.

VIRTUAL STRATEGIES

Virtual strategies could be viewed as strategies that change the natural state of a system. Once the natural state of a system is changed, the equilibria of the resultant games are regarded as virtual equilibria. Virtual strategies are artificial agents introduced to bring about improvements in a natural system. If there are infinite numbers of improvements, then there must be infinite number of strategies, hence, from Nash's theorem no virtual equilibrium would exist.

Virtual strategies could be either perfect or imperfect. The perfect virtual strategy would never fail. On the other hand an imperfect virtual strategy could fail. An imperfect virtual strategy could either be strong or weak.

A strong virtual strategy will not easily be detected and will not easily fail. A weak virtual strategy could easily be detected and could easily fail. Perfect virtual strategies lead to perfect virtual equilibria, while imperfect virtual strategies lead to imperfect virtual equilibria.

In improperly applied virtual strategies, the players assume the strategy is perfect while in actual fact it is not. Hence, the imperfect virtual equilibria is trembling. This is illustrated in Figures 1 and 2, and the following analysis.

For perfect virtual equilibrium we have probability of success P defined as:

P (Success) = 1

For imperfect virtual equilibrium we have probability of success P defined as:

P (Success) < 1

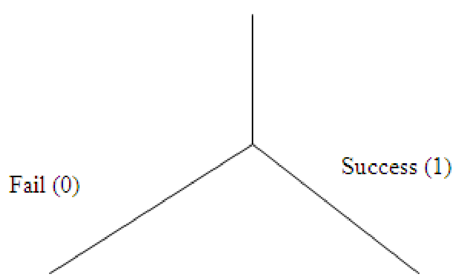

Figure 1: Perfect Virtual Equilibria.

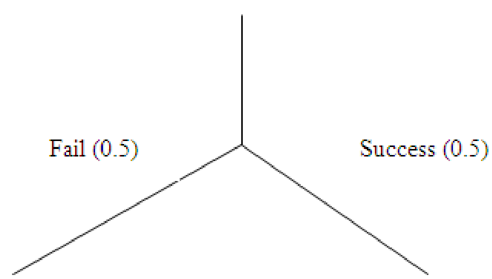

Figure 2: Imperfect Virtual Equilibria

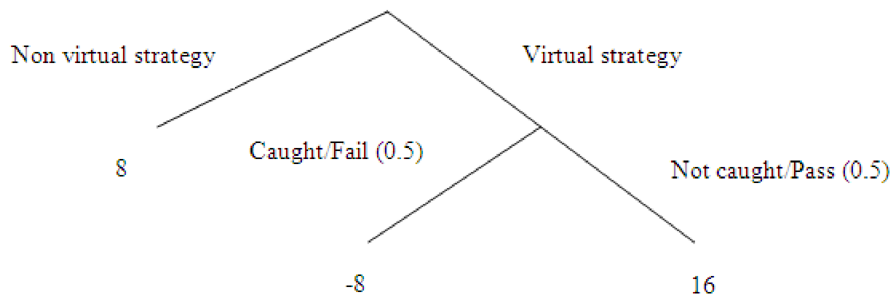

Figure 3: Virtual Strategy Construction Technique.

Figure 3 could be used to construct a 2-persons virtual game shown in Figure 4.

	Non virtual strategy	Virtual strategy
Non virtual strategy	8, 8	8, 4
Virtual strategy	4, 8	4, 4

Figure 4: Fall Simulation Game.

Assuming E(pass) = 1 in Figure. 3, then Figure 3 will change to Figure 5.

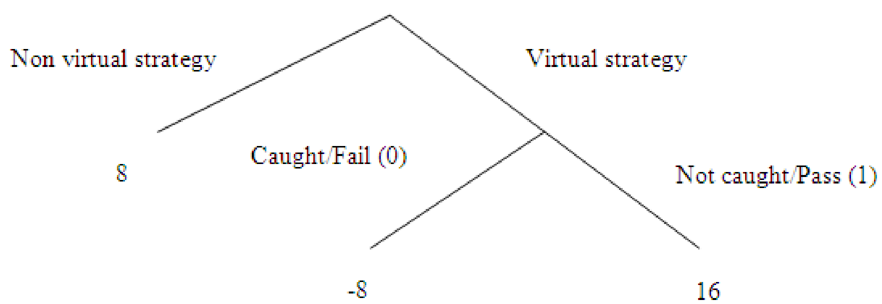

Figure 5: Virtual Strategy Construction Technique.

Figure 5 will give the payoff matrix shown in Figure 6.

	Non virtual strategy	Virtual strategy
Non virtual strategy	8, 8	8, 16
Virtual strategy	16, 8	16, 16

Figure 6: Fall Simulation Game.

A look at Figure 3 shows that if a player using the virtual strategy is actually caught the payoff would be -8, if not the payoff would be 16.

The game in Figure 3 has an imperfect virtual equilibrium and failed virtual strategy would lead to a payoff not apparent from the payoff matrix, while successful strategy would equally lead to a similar payoff, but in this case the payoff is positive.

This model could be used to model Enron's virtual strategy (doctored accounting information) which led to its downfall. Of course the virtual strategy is not perfect, hence, its eventual failure. If they had applied the strategy over a limited period, with the knowledge that it is imperfect, and tidied up gray areas in their organization with the short term gain, and then revert to zero virtual strategy, they would have been alive at the moment.

VIRTUAL EQUILIBRIA

General Cases

Consider the game in Exhibit 1 which shows two opposing players in a soccer game. Each could simulate a fall in the 18 yards box to earn a penalty (virtual strategy). For a morally sound or religious player simulating a fall would be immoral, hence, his/her strategy would be don't simulate even though it yields lower payoff. For a non religious player his strategy would be simulate.

If a morally sound and a non-religious player are playing such a game, the possible equilibrium points are circled in Exhibit 2.

	Don't Simulate	Simulate
Don't Simulate	8, 8	8, 16
Simulate	16, 8	16, 16

Exhibit 1: Fall Simulation Game.

	Don't Simulate	Simulate
Don't Simulate	8, 8	8, 16
Simulate	16, 8	16, 16

Exhibit 2: Fall Simulation Game.

If two morally sound players are playing such a game, the possible equilibrium point is circled in Exhibit 3.

	Don't Simulate	Simulate
Don't Simulate	8, 8	8, 16
Simulate	16, 8	16, 16

Exhibit 3: Fall Simulation Game.

If the two players are not religious the possible equilibrium point is circled in Exhibit 4.

	0	1
0	8, 8	8, 16
1	16, 8	16, 16

Exhibit 4: Fall Simulation Game.

For the non-religious the focal point is strategy 1, but for the religious the focal point would be strategy 0.

Consider the game in Exhibit 5. The game shows two competing athletes. Each could use drugs (a virtual strategy) but due to the possibility of detecting the drugs and the attendant penalty, the payoff when drugs are used are reduced. Obviously the equilibrium point would be point 0,0 (i.e. don't use drugs, don't use drugs).

	0 Don't use drugs	1 Use drugs
0 Don't use drugs	8, 8	8, 4
1 Use drugs	4, 8	4, 4

Exhibit 5: Drug Use Game.

Assuming no penalties for drug use, the matrix could be as shown in Exhibit 6.

This is obviously similar to the matrix in our previous game. The equilibrium point would be point 1, 1. Assuming the athlete could use an undetectable drug and there is penalty for drug use, obviously the game matrix would be as in fig. 7

	0 Don't use drugs	1 Use drugs
0 Don't use drugs	8, 8	8, 4
1 Use drugs	16, 8	16, 16

Exhibit 6: Drug Use Game.

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	0 Don't use drugs	1 Use drugs
0 Don't use drugs	8, 8	8, 4
1 Use drugs	16, 8	16, 16

Exhibit 7: Drug Use Game.

Obviously, the equilibrium points would be as in the simulation game. Assuming there is an infinite possibility of developing undetectable steroids, then there would be infinite possible strategies and the game would not have an equilibrium point. In real life interpretation, in such a situation it would be impossible to fight doping.

Consider the game in Exhibit 8. The game shows two students with two possible strategies for an examination.

	0 Don't Cheat	1 Cheat
0 Don't Cheat	8, 8	8, 16
1 Cheat	16, 8	16, 16

Exhibit 8: Cheating Game.

Each could either cheat (a virtual strategy) or not. If both students are morally sound, the possible equilibrium point is circled as shown in Exhibit 8. Hence, the focal point would be don't cheat, don't cheat. Assuming two morally deficient players, the equilibrium point would be as circled in Exhibit 9.

	0 Don't Cheat	1 Cheat
0 Don't Cheat	8, 8	8, 4
1 Cheat	16, 8	16, 16

Exhibit 9: Cheating Game.

Hence, the focal point would be cheat, cheat. Similarly, one student is morally sound while the other is not the equilibrium point would be as circled in Exhibit 10. Assuming there is no penalty for malpractice, the matrix in Exhibit 11 would be obtained.

	0 Don't Cheat	1 Cheat
0 Don't Cheat	8, 8	8, 16
1 Cheat	16, 8	16, 16

Exhibit 10: Fall Simulation Game.

	0 Don't Cheat	1 Cheat
0 Don't Cheat	8, 8	8, 4
1 Cheat	4, 8	4, 4

Exhibit 11: Cheating Game.

The obvious equilibrium point would be as circled in Exhibit 11. The focal point is therefore don't cheat, don't cheat.

In some countries for example educational institutions that are poor in enforcement of exam malpractice regulations actually find themselves recording up to 50 percent rate of cheating in exams. Institutions with very strong record of enforcement record less than 10percent rate of exam malpractice.

<u>Sub game Perfect and Trembling Virtual Equilibria</u>

Virtual games could be dynamic, and in such situations, the extensive and dynamic form representation would be used. In such situations we could have further refinement of the Nash equilibria in the nature of sub game perfect and trembling virtual equilibria.

Consider the game in Exhibit 12. Here the players have the option of either renting a crowd or not during their political campaign (Nwobi-Okoye, 2010a)

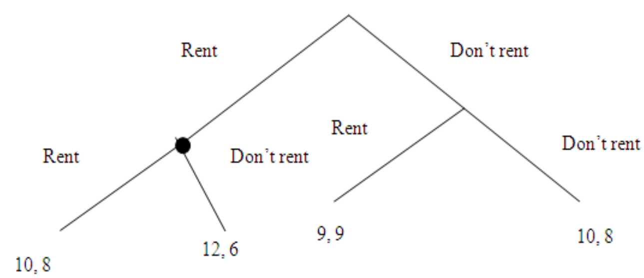

Exhibit 12: Dynamic Representation of a Virtual Game.

<u>Floating Virtual Equilibria</u>

In some situations equilibrium depends on the order of play of a game (Nwobi-Okoye, 2010b). In other words, the equilibrium outcomes depends on whether a player moved first, who started the game first (the first mover), or whether the game is played simultaneously by the players. Such situations are typical of some virtual games, such as virtual queuing games. In such situations, the extensive or dynamic form representation is not enough to tell the whole story. If we assume the order of moves to represent states of the game, there is a game associated with every state k Є K. For each state, k, there are action spaces $A_i(k)$.

Theorem: Equilibria exists in floating virtual games with finite number of states and actions.

Proof: Construct a floating strategic form in which each agent (i,k) has the payoff function of player i at state k. Since there are finite numbers of players, there are finitely many states each of whom has finite number of pure actions. From Nash's theorem, this implies that the game has Nash equilibrium.

Dynamic Virtual Equilibria

In some situations, the elements in the primary payoff matrix change when virtual strategies come with discount factors. Consider the virtual game with a virtual payoff matrix, G_n, as shown in Exhibit 13.

a_{11}-λ, b_{11}- λ	a_{12}-λ, b_{12}- λ
a_{21}-λ, b_{21}- λ	a_{22}-λ, b_{22}- λ

Exhibit 13: Virtual Game with Discount Factor.

The discount factor λ is a function of time. λ represents the cost of implementing the virtual strategy which is time dependent (i.e., the implementation cost changes with time). The equilibria of such virtual games are regarded as dynamic virtual equilibria and conform to the closed graph property of Nash equilibria (Fudenberg and Tyrole, 1991). Usually, with such games, depending on the value of λ equilibria of such games could correspond to zero virtual strategy. In order words, the use of virtual strategies may not lead to Pareto efficient outcomes in such games.

Mixed Virtual Equilibria

Assuming a service center (bank, restaurant, barbing center etc) where virtual strategy is used to induce waiting (Nwobi-Okoye, 2009) could either target the young or the elderly. The game matrix is as shown in Exhibit 14.

	1 Young	2 Old
1 Young	20, 20	40, 60
2 Old	60, 40	30, 30

Exhibit 14: Waiting Game.

Assuming two competitors x and y. the optimal strategy is a mixed strategy. That is for player x:

$20x_1 + 60x_2 = 60x_1 + 30x_2$

$20x_1 + 60(1-x_1) = 60x_1 + 30(1-x_1)$

$x_1 = 3/4$

$x_2 = 4/7$

The payoff for using mixed strategy is 37.94.

For the old, the obvious focal point would be strategy 2 which could be a CNN broadcast. For the young, the obvious focal point could be MTV (music television) broadcast.

Bayesian Virtual Equilibria

For a virtual Bayesian Game

$$\Gamma = \left\langle N, (\Theta_i), (S_i), (p_i), (u_i) \right\rangle$$

A virtual strategy profile $(s_1^*(.), s_2^*(.), ... s_n^*(.))$ is said to be a virtual Bayesian Nash Equilibrium if:

$$u_{\theta_i}(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})) \geq u_{\theta_i}(s_i, s_{-i}^*(\theta_{-i}))$$
$$\forall s_i \in S_i \;\; \forall \theta_i \in \Theta_i \;\; \forall \theta_{-i} \in \Theta_{-i} \;\; \forall i \in N.$$

As an example, assuming an upstart company (a market entrant) developed a new product (say a new brand of mobile phone) and wants to enter a market already dominated by a big competitor. The big competitor (player 1) can either be in a strong or weak position. If player 1 has developed a sophisticated technology to rival player 2's product, player 1 is said to be in a strong position. Similarly, if player 1's available technology is not as sophisticated as that of player 2, player 1 is said to be in a weak position.

Meanwhile, if player 2 enters the market, it can either use a virtual strategy (deceptive labeling, trademark, features, etc.) or may not. If player 2 enters; player 1 can either announce a rival product or if in a weak position cede. There is a 50/50 chance of player 1 being in a strong or weak position.

This game is a typical example of a Bayesian game where a virtual strategy is used. The extensive and strategic form representations of the game are shown in Exhibit 7 and 8.

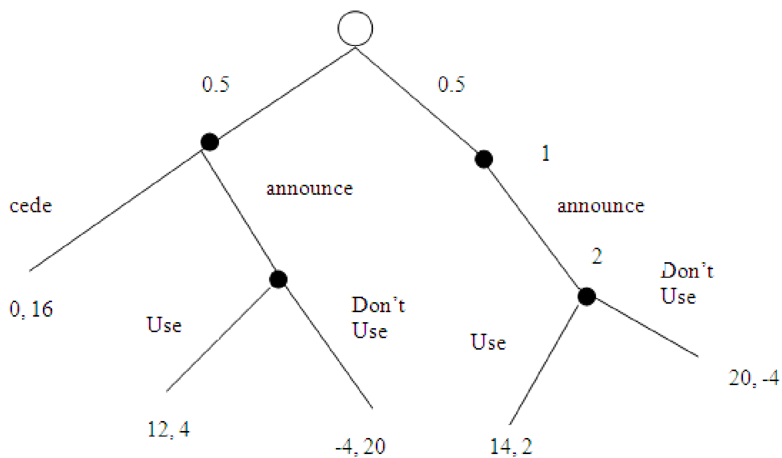

Exhibit 15: Extensive from Representation of a Bayesian Virtual Game.

The effective payoff for the non use of virtual strategy if player 1 announces is:

½(-4+20), ½ (20-4)
8, 8

The effective payoff for the use of virtual strategy for both players if player 1 announces is:
½(-14+12), ½ (2+4)
13, 3

If player 1 cedes, the effective payoff for the non use of virtual strategy is:
½(0+20), ½ (16-4)
10, 16

If player 1 cedes, the effective payoff for the use of virtual strategy is:
½(0+14), ½ (16+2)
7, 9

Hence, the strategic form representation of the game is shown in Exhibit 16.

	Don't Use	Use
announce	12, 4	12, 4
cede	12, 4	12. 4

Exhibit 16: Strategic form Representation of a Bayesian Virtual Game.

The strategic game in Exhibit 16 has equilibrium in mixed strategy. The equilibrium is known as Bayesian virtual equilibrium. If the virtual strategies are perfect we have Bayesian perfect virtual equilibrium, if not we have Bayesian imperfect virtual equilibrium.

Correlated Virtual Equilibria

For any virtual game, p (.) is a correlated virtual equilibrium if, for every player i and every virtual strategy s_i with $p(s_i)>0$,

$$\sum_{s_{-i} \in S_{-i}} p(s_{-i}|s_i) u_i(s_i, s_{-i}) \geq \sum_{s_{-i} \in S_{-i}} p(s_{-i}|s_i) u_i(s_i', s_{-i})$$
$$\forall s_i' \in S_i$$

The equation above means that player i should not be able to gain by disobeying the recommendation to play s_i if every other player obeys his recommendation.

A look at the game in Exhibit 17 shows that there are two possible strategies. One is non-virtual (NV) while the other is virtual (V). The game has three virtual equilibriums: (NV, V), (V, NV) and a mixed strategy virtual equilibrium of 3 assuming both players randomize by assigning a probability of ½ to each strategy.

	0 N	1 V
0 NV	1, 1	2, 5
1 V	5, 2	4, 4

Exhibit 17: Possible Strategies.

Assuming the players actions are correlated by a third party that assigns a strategy to each player depending on what was drawn from three cards labeled (NV,V), (V, NV) and (V,V). The probability of the three points (NV,V), (V, NV) and (V,V) occurring is 1/3 each. Hence, the value of the game expected payoff when the play is correlated is: 3(2/3) for each player. Consequently, we have a correlated virtual equilibrium.

Cooperative Virtual Payoffs/Outcomes/Points/Solutions

When virtual reality was introduced to game theory (Nwobi-Okoye, 2009; Nwobi-Okoye, 2010b), it was assumed that a virtual game is non cooperative. But this assumption is not always true, as cooperating agents could use virtual strategies. In a previous work (Nwobi-Okoye, 2010b); where virtual reality was used to increase the bargaining power in cooperative situations was briefly discussed. Such situations occur in our everyday life. When this occurs, we obtain a virtual cooperative payoff/point.

In order to illustrate the concept of virtual solution in bargaining, let's use the Nash bargaining concept. Assume a *bargaining set S* with *threat point* (u_0,v_0) is a non-empty subset of R^2 so that:

(a) $u \geq u_0$, $v \geq v_0$ for all (u,v) Є *S*;
(b) *S* is compact;
(c) *S* is convex.

The threat point (u_0,v_0) is the pair of utilities obtained by each player when they don't use virtual strategies. Each pair (u,v) in a bargaining set *S* represents a utility *u* for player I and a utility *v* for player II which the players can achieve by reaching a specific agreement when either or both players uses virtual strategy and are regarded as virtual bargaining solutions.

Fraudsters often use virtual strategies to obtain agreements and cooperation from their victims. In general, cooperative solutions obtained through virtual strategies are virtual solutions. Similarly, some organizations or institutions use virtual strategies to obtain permits and certifications from government or supervisory agencies.

CONCLUSIONS

Before I conclude, I would like to ask an open question. If you know a man that looks exactly like Tiger Woods, would you hire him for an advertisement? Is it ethical to hire him for an advertisement?

I believe I have not discussed exhaustively, equilibrium in the theory of virtual games, but their analyses have been pioneered. It is expected that a lot of further work will be done in this area, especially on dynamic Nash equilibria and cooperative virtual points.

Finally, "He who comes for equity must come with clean hands". I therefore wish to propose the axiom: "A company that uses virtual strategies must have good products".

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