Gravitational Scalar Potential Values Exterior to the Sun and Planets.

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ABSTRACT

In this paper, we compute gravitational scalar potential values along the equator and pole of the Sun and planets. Our computation differs from previous approaches in that the precise shapes (oblate spheroidal) of these astrophysical bodies are used. The values obtained diminish with increasing distance from the massive astrophysical body. A plot of the values agrees satisfactorily with well known experimental facts.

(Keywords: scalar potential, oblate spheroid, sun, planets, equator, pole)

INTRODUCTION

Prior to 1950, theoretical gravitational study was restricted almost exclusively to the fields of massive bodies of perfect spherical geometry. Since 1950, it has increasingly been recognized that the Earth, Sun, and almost all major astronomical bodies are actually spheroidal in geometry as shown in Table 1. It has also been confirmed that this geometry has experimentally measurable and physically interesting effects on the motion of test particles in their gravitational fields. These effects exist in Newton's Theory of Universal Gravitation as well as in Einstein's Theory of General Relativity [1].

It is now well known that satellite orbits around the Earth are governed by not only the simple inverse distance squared gravitational fields due to perfect spherical geometry, but are also governed by second harmonics (pole of order 3) as well as fourth harmonics (pole of order 5) of gravitational scalar potential not due to perfect spherical geometry. Therefore, towards the more precise explanation and prediction of satellite orbits around the Earth, Stern [3] and Garfinkel [4] introduced the method of quadratures for approximating the second harmonics of the gravitational scalar potential of the Earth due to its spheroidal geometry. This method was improved by O'Keefe [5].

In 1960, Vinti [6] suggested a general mathematical form of the gravitational scalar potential of the spheroidal Earth and how to estimate some of the parameters in it for use in the study of satellite orbits.

Recently, [7] the exact and complete Newton's universal gravitational fields interior and exterior to a homogenous oblate spheroidal body were derived. In this article, we use the recent expression [7] for the gravitational scalar potential exterior to a static homogenous oblate spheroid to compute approximate values of the gravitational scalar potential for the oblate spheroidal Sun and planets.

Table 1: Oblateness of Bodies in the Solar System [2].

Body	Sun	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Oblateness	9×10^{-6}	0	0	0.0034	0.006	0.065	0.108	0.03	0.026

THEORETICAL ANALYSES

The Cartesian coordinates (x, y, z) are related to the oblate spheroidal coordinates (η, ξ, ϕ) as in following equations [8];

$$x = a(1-\eta^2)^{\frac{1}{2}}(1+\xi^2)^{\frac{1}{2}}\cos\phi \quad \textbf{(1)}$$

$$y = a(1-\eta^2)^{\frac{1}{2}}(1+\xi^2)^{\frac{1}{2}}\sin\phi \quad \textbf{(2)}$$

$$z = a\eta\xi \quad \textbf{(3)}$$

where *a* is a constant parameter.

Now, consider a Cartesian coordinate system fixed at the middle of the oblate spheroidal body as shown Figure 1.

Now, let us consider the Sun and planets to be static homogenous massive oblate spheroids. It is clearly seen from Figure 1 that the x-coordinate point on the surface of the static oblate spheroidal body corresponds to its equatorial radius and its z-coordinate point on the surface correspond to its polar radius. Thus, on the equator and at the surface of the static homogenous oblate spheroidal body $(\xi = \xi_0, \eta = 0, \phi = 0)$; equation (1) becomes:

$$x_0 = a\left(1+\xi_o{}^2\right)^{\frac{1}{2}} \quad \textbf{(4)}$$

where x_0 is the equatorial radius of the body. Also, at the surface along the polar line $(\xi = \xi_0, \eta = 1)$;

$$z_0 = a\xi_0 \quad \textbf{(5)}$$

where z_0 is the polar radius of the body. The polar and equatorial radii of the Sun and planets are well known [9]. Substituting the values of the polar and equatorial radii of the Sun and planets into equations (4) and (5) and solving the equations simultaneously yields the constants ξ_0 and a for the oblate spheroidal bodies in the universe as given in Table 2.

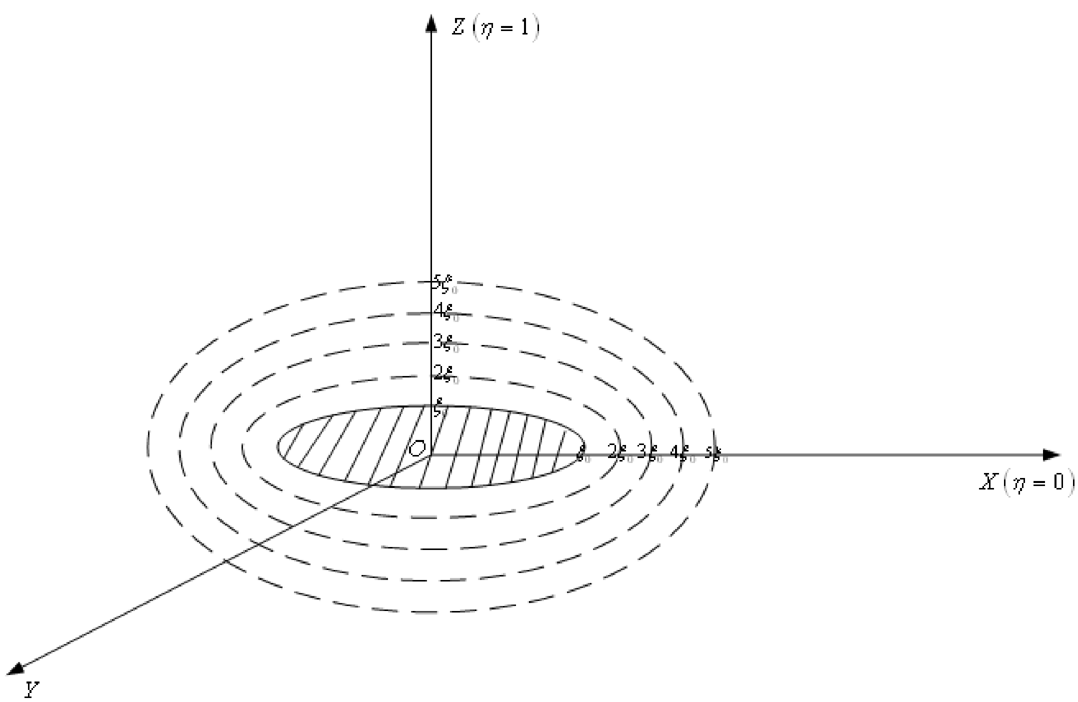

Figure 1: Static Homogenous Oblate Spheroidal Body.

Table 2: Values for the constants ξ_0 and a for the Static Homogenous Oblate Spheroidal Sun and Planets

Body	Equatorial radius(x10^3 m)	Polar Radius(x10^3 m)	ξ_0	$a(m)$
Sun	700,000	699,994	241.52	2.89829x10^6
Mercury	2,440	2,440	-	-
Venus	6,052	6,052	-	-
Earth	6,378	6,356	12.01	5.29226x10^5
Mars	3,396	3,376	09.17	3.68157x10^5
Jupiter	71,490	66,843	02.64	2.53193x10^7
Saturn	60,270	53,761	01.97	2.72899x10^7
Uranus	25,560	24,793	03.99	6.21378x10^6
Neptune	24,760	24,116	04.30	5.60837x10^6

The gravitational scalar potential exterior to a static homogeneous spheroid [7] is given as:

$$f(\eta,\xi) = B_0 Q_0(-i\xi) P_0(\eta) + B_2 Q_2(-i\xi) P_2(\eta) \quad \textbf{(6)}$$

where Q_O and Q_2 are the Legendre functions linearly independent to the Legendre polynomials P_0 and P_2 respectively. B_0 and B_2 are constants given by:

$$B_0 = \frac{4\pi G \rho_0 a^2 \xi_0}{3\Delta_1} \quad \textbf{(7)}$$

and

$$B_2 = \frac{4\pi G \rho_0 a^2}{9\Delta_2}\left[\frac{d}{d\xi} P_2(-i\xi)\right]_{\xi=\xi_0} \quad \textbf{(8)}$$

where Δ_1 and Δ_2 are defined as

$$\Delta_1 = \left[\frac{d}{d\xi} Q_0(-i\xi)\right]_{\xi=\xi_0} \quad \textbf{(9)}$$

and

$$\Delta_2 = Q_O\left[\frac{d}{d\xi} P_2(-i\xi)\right]_{\xi=\xi_0} - P_2(-i\xi)\left[\frac{d}{d\xi} Q_2(-i\xi)\right]_{\xi=\xi_0} \quad \textbf{(10)}$$

G is the universal gravitational constant. Also, it may be noted [8] that :

$$Q_0(t) = \frac{1}{2}\ln\left(\frac{t+1}{t-1}\right) \quad \textbf{(11)}$$

and

$$Q_2(t) = \frac{1}{4}\left(3t^2-1\right)\ln\left(\frac{t+1}{t-1}\right) + \frac{3}{2}t \quad \textbf{(12)}$$

Consequently, it can be shown that the Legendre functions Q_O and Q_2 can be expanded to give:

$$Q_0(-i\xi) = i\left[\xi^{-1} + \frac{1}{3}\xi^{-3} + \frac{1}{5}\xi^{-5} + \ldots\right] \quad \textbf{(13)}$$

and

$$Q_2(-i\xi) = -\frac{i}{2}\left[2\xi^{-1} + \left(\frac{1}{3}+\frac{3}{5}\right)\xi^{-3} + \left(\frac{1}{5}+\frac{3}{7}\right)\xi^{-5} + \ldots\right] \quad \textbf{(14)}$$

Thus, from equations (9) and (13) we have

$$\Delta_1 = -i\left(\frac{1}{\xi_0^{\,2}} + \frac{1}{\xi_0^{\,4}} + \ldots\right) \quad \textbf{(15)}$$

Considering only the first two terms in the expansion of Equation (15) (that is approximation of Δ_1 to the order of ξ_0^{-4}) and substituting in equation (7) gives:

$$B_0 \approx \frac{4\pi G\rho_0 a^2 \xi_0^{\,5}}{3\left(1+\xi_0^{\,2}\right)} i \qquad \textbf{(16)}$$

Similarly, approximating Δ_2 to the order of ξ_0^{-4} and substituting in Equation (8) gives:

$$B_2 \approx \frac{40\pi G\rho_0 a^2 \xi_0^{\,5}}{3\left[44\xi_0^{\,2}+\left(1+3\xi_0^{\,2}\right)\left(7+10\xi_0^{\,2}\right)\right]} i \qquad \textbf{(17)}$$

The mean density (ρ_0) for various bodies in the universe is given by [9]; the universal gravitational constant is well known to be given as; $G = 6.7x10^{-11} Nm^2 kg^{-2}$ and with the values of ξ_0 and a computed above (Table 2), we get the values for the constants B_0 and B_2 for the static homogenous oblate spheroidal Sun and planets from Equations (16) and (17) as shown in Table 3.

By considering the first two terms of the series expansion of the Legendre functions, Equations (13) and (14); Equation (6) can be written more explicitly as;

$$f(\eta,\xi) \approx \frac{B_0}{3\xi^3}\left(1+3\xi^2\right)i - \frac{B_2}{30\xi^3}\left(7+15\xi^2\right)\left(3\eta^2-1\right)i \qquad \textbf{(18)}$$

Thus we can write;

$$f(\eta,\xi) \approx \frac{B_0}{3\xi^3}\left(1+3\xi^2\right)i + \frac{B_2}{30\xi^3}\left(7+15\xi^2\right)i \qquad \textbf{(19)}$$

$$f(\eta,\xi) \approx \frac{B_0}{3\xi^3}\left(1+3\xi^2\right)i - \frac{B_2}{15\xi^3}\left(7+15\xi^2\right)i \qquad \textbf{(20)}$$

as the respective expressions for the gravitational scalar potential along the equator and pole exterior to static homogenous oblate spheroidal bodies.

Now, with the computation of the constant ξ_0 for the static homogenous oblate spheroidal Sun and planets, we can now evaluate the scalar potential along the equator and the pole at various points (multiples of ξ_0) exterior to the static homogenous oblate spheroidal Sun and planets. The results are shown in Tables 4 to 10 and Figures 2 and 3.

Table 3: Values of the constants B_0 and B_2 for the Static Homogenous Oblate Spheroidal Sun and Planets.

Body	**Mean Density,** $\rho_0(kgm^{-3})$	$B_0(Nmkg^{-1})$	$B_2(Nmkg^{-1})$
Sun	1409	$4.67961x10^{13}i$	$8.91380x10^{7}i$
Mercury	5400	-	-
Venus	5200	-	-
Earth	5500	$7.43766x10^{8}i$	$1.70123x10^{5}i$
Mars	3900	$1.13049x10^{8}i$	$4.40357x10^{5}i$
Jupiter	1300	$3.76352x10^{9}i$	$1.50951x10^{7}i$
Saturn	690	$8.76690x10^{8}i$	$5.70607x10^{6}i$
Uranus	1300	$8.41939x10^{8}i$	$1.61800x10^{6}i$
Neptune	1600	$1.06534x10^{9}i$	$1.78225x10^{6}i$

Table 4a: Gravitational Scalar Potential at Various Points along the Equator Exterior to the Static Homogenous Oblate Spheroidal Sun.

ξ	**Radial distance along the equator from center (km)**	$f(\eta,\xi)$ **along the equator (** $x10^{11} Nmkg^{-1}$ **)**
ξ_0 (241.52)	700,000	-1.9375791
$2\xi_0$ (483.04)	1,399,993	-0.9687854
$3\xi_0$ (724.56)	2,099,987	-0.6458564
$4\xi_0$ (966.08)	2,799,982	-0.4843922
$5\xi_0$ (1207.60)	3,499,976	-0.3875137
$6\xi_0$ (1449.12)	4,199,971	-0.3229281
$7\xi_0$ (1690.64)	4,899,966	-0.2767955
$8\xi_0$ (1932.16)	5,599,961	-0.2421960
$9\xi_0$ (2173.68)	6,299,956	-0.2152854
$10\xi_0$ (2415.20)	6,999,951	-0.1937568

Table 4b: Gravitational Scalar Potential at Various Points along the Pole Exterior to the Static Homogenous Oblate Spheroidal Sun.

ξ	**Radial distance along the Pole from center (km)**	$f(\eta,\xi)$ **along the pole(** $x10^{11}$ $Nmkg^{-1}$ **)**
ξ_0 (241.52)	699,994	-1.9375736
$2\xi_0$ (483.04)	1,399,990	-0.9687827
$3\xi_0$ (724.56)	2,099,985	-0.6458546
$4\xi_0$ (966.08)	2,799,980	-0.4843908
$5\xi_0$ (1207.60)	3,499,975	-0.3875126
$6\xi_0$ (1449.12)	4,199,970	-0.3229271
$7\xi_0$ (1690.64)	4,899,965	-0.2767947
$8\xi_0$ (1932.16)	5,599,960	-0.2421953
$9\xi_0$ (2173.68)	6,299,955	-0.2152847
$10\xi_0$ (2415.20)	6,999,950	-0.1937563

Table 5a: Gravitational Scalar Potential at Various Points along the Equator Exterior to the Static Homogenous Oblate Spheroidal Earth.

ξ	**Radial distance along the equator from center (km)**	$f(\eta,\xi)$ **along the equator** (**$x10^7\ Nmkg^{-1}$**)
ξ_0 (12.01)	6,378	-6.2079113
$2\xi_0$ (24.02)	12,723	-3.0985880
$3\xi_0$ (36.03)	19,075	-2.0650627
$4\xi_0$ (48.04)	25,430	-1.5486230
$5\xi_0$ (60.05)	31,784	-1.2388340
$6\xi_0$ (72.06)	38,140	-1.0323325
$7\xi_0$ (84.07)	44,495	-0.8848413
$8\xi_0$ (96.08)	50,851	-0.7742277
$9\xi_0$ (108.09)	57,207	-0.6881971
$10\xi_0$ (120.1)	63,562	-0.6193740

Table 5b: Gravitational Scalar Potential at Various Points along the Pole Exterior to the Static Homogenous Oblate Spheroidal Earth.

ξ	**Radial distance along the pole from center (km)**	$f(\eta,\xi)$ **along the pole(**$x10^7\ Nmkg^{-1}$**)**
ξ_0 (12.01)	6,356	-6.2057797
$2\xi_0$ (24.02)	12,712	-3.0975247
$3\xi_0$ (36.03)	19,068	-2.0643541
$4\xi_0$ (48.04)	25,424	-1.5480917
$5\xi_0$ (60.05)	31,780	-1.2384090
$6\xi_0$ (72.06)	38,136	-1.0319784
$7\xi_0$ (84.07)	44,492	-0.8845378
$8\xi_0$ (96.08)	50,848	-0.7739620
$9\xi_0$ (108.09)	57,204	-0.6879610
$10\xi_0$ (120.1)	63,560	-0.6191616

Table 6: Gravitational Scalar Potential at Various Points along the Equator and Pole Exterior to the Static Homogenous Oblate Spheroidal Mars.

ξ	$f(\eta,\xi)$ **along the equator** ($x10^7\ Nmkg^{-1}$)	$f(\eta,\xi)$ **along the pole(** $x10^7\ Nmkg^{-1}$ **)**
ξ_0 (09.17)	-1.2401149	-1.2328717
$2\xi_0$ (18.34)	-0.6182198	-0.6146132
$3\xi_0$ (27.51)	-0.4119197	-0.4095171
$4\xi_0$ (36.68)	-0.3088802	-0.3070788
$5\xi_0$ (45.85)	-0.2470821	-0.2456412
$6\xi_0$ (55.02)	-0.2058918	-0.2046911
$7\xi_0$ (64.19)	-0.1764735	-0.1754437
$8\xi_0$ (73.36)	-0.1544114	-0.1535109
$9\xi_0$ (82.53)	-0.1372528	-0.1364524
$10\xi_0$ (91.7)	-0.1235264	-0.1228060

Table 7: Gravitational Scalar Potential at Various Points along the Equator and Pole Exterior to the Static Homogenous Oblate Spheroidal Jupiter.

ξ	$f(\eta,\xi)$ **along the equator** ($x10^9\ Nmkg^{-1}$)	$f(\eta,\xi)$ **along the pole(** $x10^9\ Nmkg^{-1}$ **)**
ξ_0 (2.64)	-1.4968068	-1.4968068
$2\xi_0$ (5.28)	-0.7227639	-0.7184037
$3\xi_0$ (7.92)	-0.4786772	-0.4757970
$4\xi_0$ (10.56)	-0.3581710	-0.3560238
$5\xi_0$ (13.20)	-0.2862339	-0.2845140
$6\xi_0$ (15.84)	-0.2383890	-0.2369569
$7\xi_0$ (18.48)	-0.2042614	-0.2030345
$8\xi_0$ (21.12)	-0.1786879	-0.1776147
$9\xi_0$ (23.76)	-0.1588088	-0.1578550
$10\xi_0$ (26.40)	-0.1429118	-0.1420536

Table 8: Gravitational Scalar Potential at Various Points along the Equator and Pole Exterior to the Static Homogenous Oblate Spheroidal Saturn.

ξ	$f(\eta,\xi)$ **along the equator** ($x10^8\ Nmkg^{-1}$)	$f(\eta,\xi)$ **along the pole(** $x10^8\ Nmkg^{-1}$**)**
ξ_0 (1.97)	-4.8486581	-4.7999865
$2\xi_0$ (3.94)	-2.2803393	-2.2579626
$3\xi_0$ (5.91)	-1.5024497	-1.4877739
$4\xi_0$ (7.88)	-1.1221709	-1.1112275
$5\xi_0$ (9.85)	-0.8960089	-0.8872776
$6\xi_0$ (11.82)	-0.7458919	-0.7386265
$7\xi_0$ (13.79)	-0.6389317	-0.6327097
$8\xi_0$ (15.76)	-0.5588356	-0.5533945
$9\xi_0$ (17.73)	-0.4966029	-0.4917682
$10\xi_0$ (19.7)	-0.4468525	-0.4425026

Table 9: Gravitational Scalar Potential at Various Points along the Equator and Pole Exterior to the Static Homogenous Oblate Spheroidal Uranus.

ξ	$f(\eta,\xi)$ **along the equator** ($x10^8\ Nmkg^{-1}$)	$f(\eta,\xi)$ **along the pole(** $x10^8\ Nmkg^{-1}$**)**
ξ_0 (3.99)	-2.1563913	-2.1501303
$2\xi_0$ (7.98)	-1.0616053	-1.0585417
$3\xi_0$ (11.97)	-0.7056887	-0.7036545
$4\xi_0$ (15.96)	-0.5287289	-0.5272054
$5\xi_0$ (19.95)	-0.4227840	-0.4215660
$6\xi_0$ (23.94)	-0.3522299	-0.3512153
$7\xi_0$ (27.93)	-0.3018648	-0.3009952
$8\xi_0$ (31.92)	-0.2641052	-0.2633445
$9\xi_0$ (35.91)	-0.2347441	-0.2340680
$10\xi_0$ (39.9)	-0.2112593	-0.2106508

Table 10: Gravitational Scalar Potential at Various Points along the Equator and Pole Exterior to the Static Homogenous Oblate Spheroidal Neptune .

ξ	$f(\eta,\xi)$ **along the equator** **(** $x10^8\,Nmkg^{-1}$ **)**	$f(\eta,\xi)$ **along the pole(** $x10^8\ Nmkg^{-1}$ **)**
ξ_0 (4.3)	-2.5243240	-2.5179499
$2\xi_0$ (8.6)	-1.2453932	-1.2422650
$3\xi_0$ (12.9)	-0.8281919	-0.8261137
$4\xi_0$ (17.2)	-0.6206005	-0.6190438
$5\xi_0$ (21.5)	-0.4962792	-0.4950345
$6\xi_0$ (25.8)	-0.4134749	-0.4124379
$7\xi_0$ (30.1)	-0.3543600	-0.3534714
$8\xi_0$ (34.4)	-0.3100383	-0.3092608
$9\xi_0$ (38.7)	-0.2755733	-0.2748822
$10\xi_0$ (43.0)	-0.2480054	-0.2473836

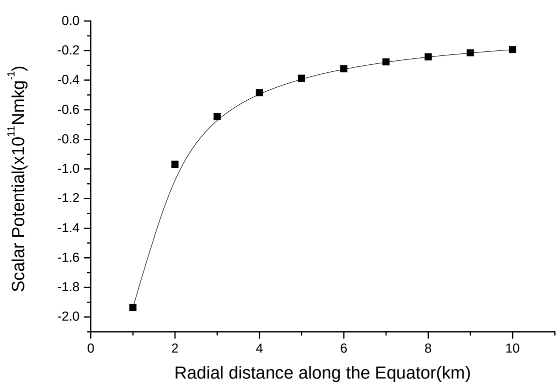

Figure 2a: Gravitational Scalar Potential at Various Points along the Equator Exterior to the Static Homogenous Oblate Spheroidal Sun.

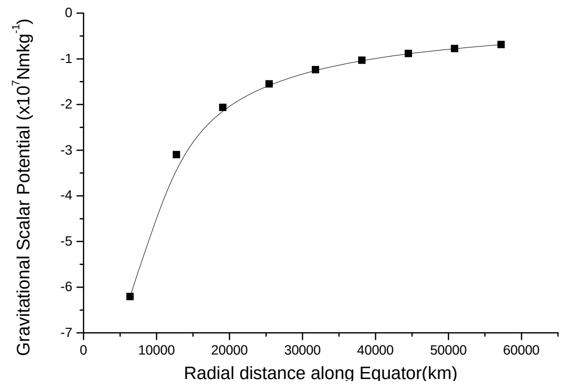

Figure 3a: Gravitational Scalar Potential at Various Points along the Equator Exterior to the Static Homogenous Oblate Spheroidal Earth.

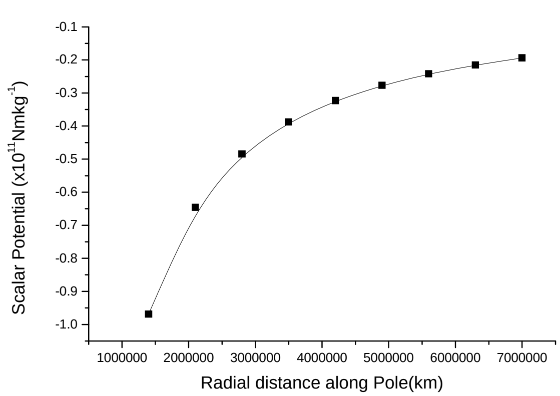

Figure 2b: Gravitational Scalar Potential at Various Points along the Pole to the Static Homogenous Oblate Spheroidal Sun.

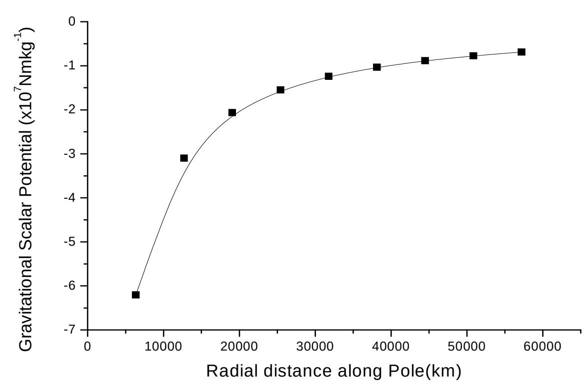

Figure 3b: Gravitational Scalar Potential at Various Points along the Pole Exterior to the Static Homogenous Oblate Spheroidal Earth.

REMARKS AND CONCLUSION

It is deduced from equation (18) that:

$$\lim_{\xi \to \infty} f\left(\eta, \xi\right) = 0 \qquad \textbf{(21)}$$

This shows that the scalar potential diminishes as one moves away from the massive oblate spheroid. We can conveniently conclude from our computations (see Tables 4 to 10 and Figures 2 and 3) above that the theoretical values of the gravitational scalar potential exterior to the static homogenous oblate spheroidal Sun and planets decreases sharply in magnitude as we move away from the surface of the Sun and planets along the equator and the pole. The exponential decrease is similar to what is obtained when the Sun and planets are considered to be static homogenous spheres. Our computations agree satisfactorily with the experimental fact that the gravitational scalar potential exterior to any regularly shaped object has maximum magnitude on the surface of the body and decreases to zero at infinity.

The immediate consequence of the results obtained above is that the exact shape of the planets and Sun was used to obtain the gravitational scalar potential on the surface at the pole and equator. Thus, instead of using the values obtained by considering the Sun and planets as homogenous spheres, our experimentally convenient values obtained can now be used. The door is now open for the computation of values for various gravitational phenomena exterior to the static homogenous oblate spheroidal Sun and planets along the equator and pole. Some of these phenomena include gravitational length contraction and time dilation.

For more accurate results, additional terms of the series expansion of the Legendre polynomials can be included. Also, our analysis in this article can be expanded to evaluate the values of scalar potential at any point exterior to the oblate spheroid. The results obtained in this article can be compounded with the correction to Scalar potential of these massive bodies due to their rotation [10].

Recently, Ioannis and Michael [10] proposed the Sagnac interferometric technique as a way of detecting corrections to the Newton's gravitational scalar potential exterior to an oblate spheroid. If this technique is developed in the near future, the theoretical study in this article will be confirmed experimentally.

REFERENCES

1. Howusu, S.X.K and E.F. Musongong. 2005. "Newton's Equations of Motion in the Gravitational Field of an Oblate Mass". *Galilean Electrodynamics*. 16(5):97-100.

2. Solar System Data. 2008. http://hyperphysics.phy_astr.gsu.edu/hbase/solar/soldata3.html

3. Stern, T.E. 1957. "Theory of Satellite Orbits". *Astronomical Journal*. 62(96).

4. Garfinkel, B. 1958. "Problem of Quadratures". *Astronomical Journal*. 63(88).

5. O'Keefe. 1958. "Improved Methods on Quadratures". *Science*. 128(568).

6. Vinti, J.P. 1960. "New Approach in the Theory of Satellite Orbits". *Physics Review Letters.* 3(1):8.

7. Howusu, S.X.K. 2005. "Gravitational Fields of Spheroidal Bodies-Extension of Gravitational Fields of Spherical Bodies". *Galilean Electrodynamics*. 16(5):98-100.

8. Arfken, G. 1995. *Mathematical Methods for Physicists, 5th edition*. Academic Press: New York, NY. 467-469.

9. Redmond, W.A. 2008. "The Solar System". *Microsoft Student Encarta*. Microsoft Corporation: Redmond, WA.

10. Ioannis, I.H. and Micheal, H. 2008. "Detection of the Relativistic Corrections to the Gravitational Potential using a Sagnac Interferometer". *Progress in Physics*. 3:3-8

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