Mathematical Modeling for Understanding of Politics in the Presence of Violence: Contemporary Nigerian Politics in Focus

Kalabu Salisu Ahmad[1*]; Abdullahi Mohammed Auwal[1]; Yusuf Bala[1]; and Huzaifa Aliyu Babando[2]

[1]Department of Mathematics and Statistics, Federal Polytechnic Bauchi, Nigeria.
[2]Department of Mathematics, Modibbo Adama University, Yola, Nigeria.

E-mail: skalabuahmad@fptb.edu.ng*
Telephone : +2348034088553

ABSTRACT

While the entire world canvasses towards democratic development in governance, many countries in the world today, especially in the African continent, do not record significant progress in that regard due to presence of violence in their political systems. There has been continued scientific descriptive efforts from individuals and the Nigerian government for instance, to provide more understanding of our political dynamics in cognizance of the problem of related violence in order to bring about elimination of the problem, but all the efforts this work came across fail to present the situation in a mathematical modeling view framework.

This paper proposes a mathematical model using a deterministic system of ordinary differential equations in order to provide deeper understanding of the problem of violence conflated with our political system in Nigeria and ways of controlling it for meaningful political development in the country. Analytically, it has been shown that the system is well posed.

Equilibrium points of the system have been obtained and conditions for both local and global asymptotic stabilities of the trivial fixed point have been established and emphasized by the computed basic reproduction number. A sub-model has been deduced from the main model and analyzed for local and global asymptotic stability of the trivial steady state. It is concluded that to curtail the problem of violence associated with our political dynamics for more peaceful living as well as meaningful socio-political development in Nigeria, more collaborative approaches need to be put in place to control the problem of violence.

(Keywords: mathematical model, politics, violence, equilibrium points, stability analysis, basic reproduction number, Nigeria)

INTRODUCTION

Politics is conceptualized as the ongoing non-violent negotiation of competing rights and interests, and the overall aim of liberalism is to remove violence from the political process (Hutchings and Frazer, 2011). Violence is regarded as the use of physical force, power , threats, or weapons against one-self, another person, a group, or community that either results in, or has a high likelihood of resulting in threat or psychological harm, deprivations, injury or death World Health Organization (WHO, 2002). Political Violence (be it pre-electoral or post-electoral), is referred to as the use of threat, power, or weapons by individual(s) in a political class against another person(s) and/or properties, intended to harm or even kill person(s), destroy properties for political reason towards injecting fear in person(s) in order to change the fair political processes (Ikyase and Egberi, 2015).

Numerous countries of the world today are going towards adopting political systems of governance and moving away from military rule or dictatorships, but unfortunately violence is retarding the acceleration of that political advancement. Violence is now eating deep into our social institutions precisely political classes and thereby bringing about human rights and socio-political deprivations in many nations especially Nigeria and other African countries. For instance (Benjamin, 2014) reported that since 1900, forms of conflict and political violence have likely killed over 100 million people mostly civilians, world over.

Prince and Luke (2021) studied and explored how the prevalence of political violence has constrained Nigeria's democracy through data generation and analysis concerning democracy and political violence and found out that the prevalence of political violence is fundamentally an attribute of vested interests of the elite people in the political class.

Mustapha and Yahaya (2021) examined the inception of electoral violence and its resultant effects on voter turnout in the 2019 general elections in Nigeria, with focusing discussion on the role of political parties. Their electoral data review shows declination in voter turnout (from 69% in 2003 to 35% in 2019). The study ascribes its prevalence to factors including the political parties (both incumbent and opposition) and politicians that use violence as an electoral strategy. The study builds on theories of voter mobilization in and advanced and emerging democracies to account for the relationship between, political parties, electoral violence, and voter turn-out. They concluded that in the absence of enduring party-voter relations in Nigeria, political parties, and politicians alike, resort to vote buying, mobilization of political thugs, and in other times, deployment of state coercive apparatus to intimidate opponents all of which resulted into electoral irregularities that has the potentiality to instigate electoral violence.

Christian (2021) examined the effect of hate speech on election violence in Nigeria during the 2011, 2015, and 2019 presidential elections by using a mixed methods approach with qualitative dominance and came up with the finding that an entrenched culture of hate speech is an oft-neglected major driver of election violence in Nigeria. The researcher concluded that the implementation of existing anti-hate speech laws provides an opportunity for protecting the rights of minority groups, promoting political inclusion, and preventing election violence in Nigeria and beyond.

Anweting and Ogar (2018) investigated the inception of Nigeria's political violence and assessed its consequences on national development and obtained that political violence dissuades domestic and foreign investment in our economy, triggers government loss of revenue and resources, and results in the election of unqualified representatives to the expense of the nation's economic and social progressive development. The research proposed reducing wages paid to political office holders, and politicians should strive to achieve national unity instead of growing religious and ethnic tensions and equitable national resources distribution by government.

Ibeogu and Joseph (2015) sought to establish the causes of political violence in Nigerian democratic practices by adopting elite theory as a theoretical framework of analysis. The study established that greed and struggle for supremacy between godfathers and godsons have often opened up to political violence in the nation's party system. In order to avoid problems of having incompetent leaders, decay of the country's polity and destruction of lives and properties, the identified factors have to be addressed through avoidance of show of ethnicity and religion as pre-requisite when it comes to electing candidates for public and political positions.

Sampson (2021) studied Nigerian State and electoral violence, with focus on why the violence that emanated from the recently concluded 2019 presidential general election in Nigeria and identified that Nigerian elections have always been recognized by hate speeches, media war among contenders, politically motivated killings and assassinations, intimidations, victimizations, hijack of electoral materials, and destruction of campaign billboards and property. He used secondary data and adopted the Marxist theoretical framework to help the study to analyze the relationship between classes within the Nigerian state, and how that relationship impacts on the dynamics of the struggle to access the power of the state. The research disclosed that the Nigeria State has been failing to address poverty and unemployment as the root causes of electoral violence and hence failed in its primary responsibility enshrined in section 14 sub-section 2(b) of the 1999 constitution of Nigeria. He called on government to sincerely tackle poverty and unemployment through educational and economic empowerment programs that would bring about socio-economic development in order to control the problems of political violence.

It is clearly observed that from the literature presented, that amongst measures put in place to address the problem of violence conflated with our political practices in Nigeria, none has actually come up with a compartmental mathematical model showing the dynamics of

political parties in Nigeria in the presence of violence. Apart from other clear indicators, many other theories reveal that our politics in Nigeria is conflated with violence. For instance, Dahrendorf's macro conflict theory (conflict or consensus) of seeing the order in society as a product of coercion of high class people or elites (Adetayo, 2021), political parties and other social organizations provide thugs with funds, weapons, and logistics to perpetrate political violence (Human Rights Watch, 2007). Therefore, we employ the use of deterministic system of ordinary differential equations to model the dynamics of two political parties in Nigeria while taking into cognizance the presence of violence or violent persons in order to suggest way(s) of controlling the vice as (Priscilla, 2020) used system of differential equations to model the dynamics of two political parties, and (Charles and Dominic, 2014) used system of ordinary differential equations and modeled the spread of violence.

Modeling the dynamics of political parties in Nigeria is presumed to be difficult for the presence of multi-parties; however, it is a clear fact that in most countries worldwide there has always been two major political parties and that makes it difficult for other minor parties to thrive and compete with the major ones but ending up collaborating with either of them (Arvind, 2012) therefore two political parties are considered in the model. The paper has been prepared in six sections include an introduction, model formulation, dynamical description of the model, model analysis, results, discussion of results, and finally conclusions.

THE MODEL FORMULATION

In the efforts to formulate the model variables and parameters are identified and defined in Table 1 and assume thus: the population under study comprises five epidemiological sub-compartments, the target population is homogeneously mixed, new individuals are born susceptible (into the susceptible compartment), and persons exit the system through natural death and inactiveness due to age.

Table 1: State Variables and Parameters of the Model

Variables/Parameters	Description
$S(t)$	Is the number of susceptible persons at active voting age at time t
$P(t)$	Is the number of members of political party P at time t
$Q(t)$	Is the number of members of political party Q at time t
$V(t)$	Is the number of violent persons at time t
$R(t)$	Is the of persons undergoing rehabilitation/treatment programs
$N(t)$	Is the number of total population at time t
τ	Is maturity recruitment rate of the population
π	Is natural exiting rate (natural death and aging rate)
σ	Is induced death rate due to violence perpetration in V class
ρ	Is induced death rate due to victimization in R class
α_1	Is the rate at which susceptible ones register with party P
α_2	Is the rate at which susceptible ones register with party Q
β_1	Is decamping rate from party Q to party P
β_2	Is decamping rate from party P to party Q
ε_1	Is convincement rate of violent persons to join party P
ε_2	Is convincement rate of violent persons to join party Q
μ	Is the rate at which susceptible persons become violent
θ	Is the rate at which violent persons enter rehabilitation class
ω	Is the rate at which violent politicians from party P enter rehabilitation class
γ	Is the rate at which violent politicians from party Q enter rehabilitation class
λ	Is the rate at which rehabilitated persons become susceptible again

The Model Flow Diagram

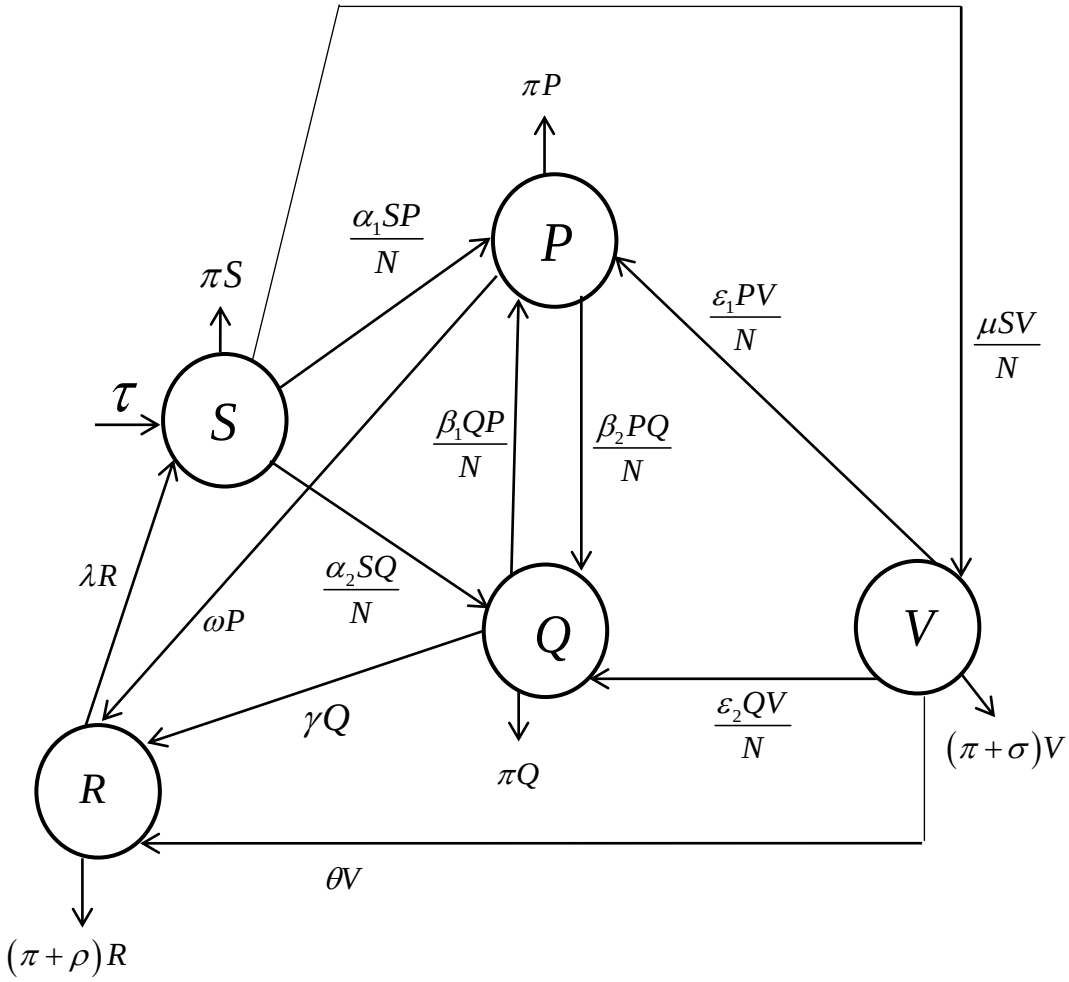

Figure 1: The Model Flow Diagram.

The Model Equations

The model equations are hereby presented below:

$$S' = \tau + \lambda R - \frac{\alpha_1 SP}{N} - \frac{\alpha_2 SQ}{N} - \frac{\mu SV}{N} - \pi S \quad \textbf{(1)}$$

$$P' = \frac{\alpha_1 SP}{N} + \frac{\varepsilon_1 PV}{N} + \frac{\beta_1 PQ}{N} - \frac{\beta_2 QP}{N} - \pi P - \omega P \quad \textbf{(2)}$$

$$Q' = \frac{\alpha_2 SQ}{N} + \frac{\varepsilon_2 QV}{N} + \frac{\beta_2 QP}{N} - \frac{\beta_1 PQ}{N} - \pi Q - \gamma Q \quad \textbf{(3)}$$

$$V' = \frac{\mu SV}{N} - \frac{\varepsilon_1 PV}{N} - \frac{\varepsilon_2 QV}{N} - \theta V - (\pi + \sigma)V \quad \textbf{(4)}$$

$$R' = \omega P + \gamma Q + \theta V - \lambda R - (\pi + \rho)R \quad \textbf{(5)}$$

where,

$$N(t) = S(t) + P(t) + Q(t) + V(t) + R(t) \quad \textbf{(6)}$$

subject to the initial conditions,

$$S(0) = S_0, P(0) = P_0, Q(0) = Q_0, V(0) = V_0, R(0) = R_0 \quad \textbf{(7)}$$

DYNAMICAL DESCRIPTION OF THE MODEL

The system's dynamic description is thus: S compartment increases by birth at the rate τ and re-susceptibility rate λ while it decreases as a result of outflow of persons joining parties P and Q at the rates α_1 and α_2 respectively, outflow of persons joining violence class at the rate μ, and also due to natural exiting rate of the system π. P Compartment is empowered by inward movements from S, Q and V at the rates α_1, β_1 and ε_1 respectively while it is depopulated by natural exiting rate, outward movement to Q at a defection rate β_2 and joining R at rehabilitation rate ω.

Q compartment is energized by inflow from S, P and V at the rates α_2, β_2 and ε_2 respectively but decreases by the natural exiting rate, defection to party P effect at the rate β_1 and joining rehabilitation at the rate γ. V is populated by inward movement of susceptible persons at the rate μ but depopulated by outward movements to parties P and Q at the rates ε_1 and ε_2 respectively, natural exiting rate π and induced death rate ρ and also due to joining rehabilitation at the rate θ.

R compartment increases by the inflow of persons from the two political parties P, Q and violent classes at the rates ω, γ and θ respectively but decreases by natural exiting rate π, victimization induced death rate ρ and re-susceptibility rate λ.

The Model Analysis

Theorem 1: Solution for the model equations (1) to (5) exists as it is bounded in the region,

$$\Omega = \left\{ (S, P, Q, V, R) \in \Re_+^5 : N \le \frac{\tau}{\pi} \right\}.$$

Proof: This follows from (Hethcote, 2000).

Recall that (6) is:

$$N(t) = S(t) + P(t) + Q(t) + V(t) + R(t)$$

$$\frac{d}{dt} N(t) = S'(t) + P'(t) + Q'(t) + V'(t) + R'(t)$$

$$\frac{dN}{dt} = \tau - \pi N - \sigma V - \rho R$$

$$\frac{dN}{dt} \le \tau - \pi N$$

$$\int \frac{1}{(\tau - \pi N)} dN \leq \int dt \quad \textbf{(8)}$$

Let $Y = (\tau - \pi N)$ **(9)**

so that $dN = -\frac{1}{\pi} dY$ **(10)**

substituting (9) and (10) into (8),

$$-\frac{1}{\pi}\int \frac{1}{Y} dY \leq \int dt \quad \text{or} \quad -\frac{1}{\pi}\ln Y \leq t + c \quad \text{or} \quad \ln(Y) \geq -\pi(t + c)$$

$$Y \geq e^{-\pi(t+c)} \quad \textbf{(11)}$$

substituting (9) into (11) we have,

$$\tau - \pi N \geq e^{-\pi(t+c)}$$

so that $N \leq \frac{\tau}{\pi} - \frac{e^{-\pi(t+c)}}{\pi}$

taking limit gives,

$$\lim_{t\to\infty} N \leq \lim_{t\to\infty}\left(\frac{\tau}{\pi} - \frac{e^{-\pi(t+c)}}{\pi}\right)$$

$$N \leq \frac{\tau}{\pi} \quad \textbf{(12)}$$

$\Omega = \Omega \subset \Re_+^5$ with $\Omega = \left\{(S, P, Q, V, R) \in \Re_+^5 : N \leq \frac{\tau}{\pi}\right\}$ for all time, $t > 0$.

Since the system is bounded as $0 \leq N \leq \frac{\tau}{\pi}$, hence it has an existing solution.

Equilibrium Points of the Main Model

The system has four (4) different equilibrium points for there are four different possibilities in the system thus:

If none of the parties as well as the violence exist, then from equations (1) to (5), and if there is no change in the classes (as at steady state) we have:

$$E_1 = (S, P, Q, V, R) = \left(\frac{\tau}{\pi}, 0, 0, 0, 0\right) \quad \textbf{(13)}$$

This is a possibility in political science, a situation called nonpartisan politics (sometimes when parties are under legal restrictions) where candidates are eligible for public office based on merit and capabilities. For instance, there were not political parties in Pakistan from 2001 – 2008 (Arvind, 2012). This equilibrium state is unstable whenever at least a party exists. If only party P exists and in the absence of violence, then from equation (2), and if there is no change in the classes (as at steady state) we have:

$$E_2 = (S, P, Q, V, R) = \left(\frac{\tau}{\pi}, 1 - \frac{\alpha_1}{(\pi + \omega)}, 0, 0, 0 \right) \quad \textbf{(14)}$$

E_2 exists if $(\pi + \omega) > \alpha_1$.

If only party Q exists and in the absence of violence, then from equation (3), and if there is no change in the classes (as at steady state) we have:

$$E_3 = (S, P, Q, V, R) = \left(\frac{\tau}{\pi}, 0, 1 - \frac{\alpha_2}{(\pi + \gamma)}, 0, 0 \right) \quad \textbf{(15)}$$

and E_3 exists if $(\pi + \gamma) > \alpha_2$.

Thus (14) and (15) are possibilities in political science, a situation called one-party system or where there is only one dominant-party in a political system. For instance, People's Action Party in Singapore and African National Congress in South-Africa (Arvind, 2012). If both the two parties exist (and in the presence of violence), then from equations (1) to (5), and if there is no change in the classes (as at steady state) we have:

$$E_4 = (S, P, Q, V, R) = (S^*, P^*, Q^*, V^*, R^*) \quad \textbf{(16)}$$

where,

$$\left.\begin{aligned}
S^* &= \frac{(\tau + \lambda R^*) N^*}{(\alpha_1 P^* + \alpha_2 Q^* + \mu V^* + \pi N^*)} \\
P^* &= \frac{(\pi + \gamma) N^* - (\alpha_2 S^* + \varepsilon_2 V^*)}{(\beta_2 - \beta_1)} \\
Q^* &= \frac{(\pi + \omega) N^* + \varepsilon_1 V^* - \alpha_1 S^*}{(\beta_2 - \beta_1)} \\
V^* &= \frac{\{\varepsilon_1 (\pi + \gamma) - (\theta + \pi + \sigma)(\beta_2 - \beta_1)\} N^* - \{(\mu S^* + \varepsilon_2 Q^*)(\beta_2 - \beta_1) + \varepsilon_1 \varepsilon_2 S^*\}}{\varepsilon_1 \varepsilon_2} \\
R^* &= \frac{\omega P^* + \gamma Q^* + \theta V^*}{(\lambda + \pi + \rho)}
\end{aligned}\right\}$$

with $(\beta_2 - \beta_1) > 0$ or $\beta_2 > \beta_1$.

Equation (16) is called endemic or interior equilibrium point, which is unstable if one or two political part(ies)y and violence do not exist.

Stability analysis of E_1

Theorem 2: The system is locally asymptotically stable if all the eigenvalues are real and negative; this holds if the following conditions are satisfied:

i. $\alpha_1 < (\pi + \omega)$

ii. $\alpha_2 < (\pi + \gamma)$

iii. $\mu < (\theta + \pi + \sigma)$

Proof: This follows from (Baba, Oluwole and Mohammed, 2016).

Jacobian matrix of equations (1) to (5) evaluated at (13) gives:

$$J_{E_1} = \begin{pmatrix} -\pi & -\alpha_1 & -\alpha_2 & -\mu & \lambda \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & c & 0 \\ 0 & \omega & \gamma & \theta & -(\pi + \lambda + \rho) \end{pmatrix}$$

For the eigenvalues g_i we use characteristics equation $\left|J_{E_1} - g_i I\right| = 0$ where $i = 1,2,3,4,5$ and I is identity matrix.

$$\left|J_{E_1} - g_i I\right| = \begin{pmatrix} -\pi - g_1 & -\alpha_1 & -\alpha_2 & -\mu & \lambda \\ 0 & a - g_2 & 0 & 0 & 0 \\ 0 & 0 & b - g_3 & 0 & 0 \\ 0 & 0 & 0 & c - g_4 & 0 \\ 0 & \omega & \gamma & \theta & -(\pi + \lambda + \rho) - g_5 \end{pmatrix} = 0$$

$g_1 = -\pi$ (< 0), $g_2 = a$, $g_3 = b$, $g_4 = c$ and $g_5 = -(\pi + \lambda + \rho)$ (< 0) where $a = \alpha_1 - (\pi + \omega)$,

$b = \alpha_2 - (\pi + \gamma)$ and $c = \mu - (\theta + \pi + \sigma)$.

$g_2 = a = \alpha_1 - (\pi + \omega) < 0$ if $\alpha_1 < (\pi + \omega)$ **(17)**

$g_3 = b = \alpha_2 - (\pi + \gamma) < 0$ if $\alpha_2 < (\pi + \gamma)$ **(18)**

and $g_4 = c = \mu - (\theta + \pi + \sigma) < 0$ if $\mu < (\theta + \pi + \sigma)$ **(19)**

Theorem 3: The fixed point $E_1 = \left(\frac{\tau}{\pi}, 0, 0, 0, 0\right)$ is a global asymptotic stable equilibrium point of the system if and only if the following conditions hold:

i. $\alpha_1 P\left(1-\frac{S}{N}\right)\geq\left(\frac{\beta_1 QP}{N}+\frac{\varepsilon_1 PV}{N}\right)$

ii. $\alpha_2 Q\left(1-\frac{S}{N}\right)\geq\left(\frac{\beta_2 PQ}{N}+\frac{\varepsilon_2 QV}{N}\right)$

Proof: This follows from (Carlos, Zhilan and Wenzhang, 2001 and Salisu *et al*, 2021).

$$\frac{dX}{dt}=T(X,I)$$
$$\frac{dI}{dt}=G(X,I), G(X,0)=0,$$

where $X=(S,R)\in\Re^2$, $I=(P,Q,V)\in\Re^3$, $T(X,0)=\left(\frac{\tau}{\pi},0,0,0\right)=E$

and $A_{E_1}=\begin{pmatrix}\alpha_1-(\pi+\omega) & 0 & 0\\ 0 & \alpha_2-(\pi+\gamma) & 0\\ 0 & 0 & \mu-(\theta+\pi+\sigma)\end{pmatrix}$ which the off-diagonal entries are non-negative.

Now $G\left(X,I\right)=\begin{bmatrix}G_1(X,I)\\ G_2(X,I)\\ G_3(X,I)\end{bmatrix}=\begin{bmatrix}\alpha_1 P\left(1-\frac{S}{N}\right)-\left(\frac{\beta_1 QP}{N}+\frac{\varepsilon_1 PV}{N}\right)\\ \alpha_2 Q\left(1-\frac{S}{N}\right)-\left(\frac{\beta_2 PQ}{N}+\frac{\varepsilon_2 QV}{N}\right)\\ \mu V\left(1-\frac{S}{N}\right)\end{bmatrix}$ **(20)**

Since at initial ($0\leq S\leq N$) and $E=\left(\frac{\tau}{\pi},0,0,0\right)$, $G\left(X,I\right)=0$. Hence the equilibrium point is globally asymptotically stable if $\alpha_1 P\left(1-\frac{S}{N}\right)\geq\left(\frac{\beta_1 QP}{N}+\frac{\varepsilon_1 PV}{N}\right)$ and $\alpha_2 Q\left(1-\frac{S}{N}\right)\geq\left(\frac{\beta_2 PQ}{N}+\frac{\varepsilon_2 QV}{N}\right)$ so that $G(X,I)\geq 0$.

Basic Reproduction Number (BRN)

Theorem 4: BRN $\Re_0=\rho\left(FT^{-1}\right)$ which is the maximum or threshold, $\left(\rho\right)$ of the eigenvalues of the resulting matrix, $\left(FT^{-1}\right)$.

Proof: This follows from (Driessche and Watmough, 2002). From equations (2), (3) and (4) we have,

$$F_i = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} = \begin{pmatrix} \frac{\alpha_1 SP}{N} + \frac{\varepsilon_1 PV}{N} + \frac{\beta_1 PQ}{N} \\ \frac{\alpha_2 SQ}{N} + \frac{\varepsilon_2 QV}{N} + \frac{\beta_2 QP}{N} \\ \frac{\mu SV}{N} \end{pmatrix}$$

$$F = \frac{\partial F_i}{\partial x_i} = \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \mu \end{pmatrix}$$

$$T_i = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} (\pi + \omega) P \\ (\pi + \gamma) Q \\ (\theta + \pi + \sigma) V \end{pmatrix}$$

$$T = \frac{\partial T_i}{\partial x_i} = \begin{pmatrix} (\pi + \omega) & 0 & 0 \\ 0 & (\pi + \gamma) & 0 \\ 0 & 0 & (\theta + \pi + \sigma) \end{pmatrix}$$

where $i = 1, 2, 3$.

$$T^{-1} = \begin{pmatrix} \frac{1}{(\pi + \omega)} & 0 & 0 \\ 0 & \frac{1}{(\pi + \gamma)} & 0 \\ 0 & 0 & \frac{1}{(\theta + \pi + \sigma)} \end{pmatrix}$$

$$FT^{-1} = \begin{pmatrix} \frac{\alpha_1}{a} & 0 & 0 \\ 0 & \frac{\alpha_2}{b} & 0 \\ 0 & 0 & \frac{\mu}{c} \end{pmatrix}$$

$$\Re_0 = \max . \left\{ \frac{\alpha_1}{a}, \frac{\alpha_2}{b}, \frac{\mu}{c} \right\}.$$

The system escalates into endemic situation of the problem if $\Re_0 > 1$ and declines if $\Re_0 < 1$.

For the BRN to be kept less than 1, these should be achieved: $\alpha_1 < a$ (if it is the threshold) or $\alpha_2 < b$ (if it is the threshold) or $\mu < c$ (if it is the threshold)

where $a = (\pi + \omega)$, $b = (\pi + \gamma)$ and $c = (\theta + \pi + \sigma)$.

Deduced Violence-Free Sub-Model

By setting $V = 0$ in equations (1) to (3) and neglecting rehabilitation (since there is no violence) we have:

$$S' = \tau - \frac{\alpha_1 SP}{N} - \frac{\alpha_2 SQ}{N} - \pi S \quad \textbf{(21)}$$

$$P' = \frac{\alpha_1 SP}{N} - \frac{\beta_1 PQ}{N} - (\pi + \omega)P \quad \textbf{(22)}$$

$$Q' = \frac{\alpha_2 SQ}{N} + \frac{\beta_2 QP}{N} - \frac{\beta_1 PQ}{N} - (\pi + \gamma)Q \quad \textbf{(23)}$$

where,

$$N(t) = S(t) + P(t) + Q(t) \quad \textbf{(24)}$$

subject to the initial conditions,

$$S(0) = S_0, P(0) = P_0, Q(0) = Q_0 \quad \textbf{(25)}$$

It can simply be deduced from (17) and (18) that the sub-system is locally asymptotically stable if: $\alpha_1 < (\pi + \omega)$ (if it is the threshold) or $\alpha_2 < (\pi + \gamma)$ (if it is the threshold). Similarly (20) becomes:

$$G(X, I) = \begin{bmatrix} G_1(X, I) \\ G_2(X, I) \end{bmatrix} = \begin{bmatrix} \alpha_1 P\left(1 - \frac{S}{N}\right) - \left(\frac{\beta_1 QP}{N}\right) \\ \alpha_2 Q\left(1 - \frac{S}{N}\right) - \left(\frac{\beta_2 PQ}{N}\right) \end{bmatrix}$$

which is globally asymptotically stable if:

$\alpha_1\left(1 - \frac{S}{N}\right) \geq \left(\frac{\beta_1 Q}{N}\right)$ and $\alpha_2\left(1 - \frac{S}{N}\right) \geq \left(\frac{\beta_2 P}{N}\right)$ so that $G(X, I) \geq 0$ as $0 \leq S \leq N$.

RESULTS

A mathematical model has been proposed and it has been established that the model has existing solution since it is bounded. Equilibrium points, local and global stabilities of the parties-and-vice-free steady state of the main model have been obtained and BRN has also been established and used to emphasize the stability of the trivial steady state of the system of the main model. Violence-free sub-model has been deduced from the main model and also, local and global asymptotic stabilities of the sub-system have been analyzed.

DISCUSSION OF THE RESULTS

The (main) model can be regarded as being epidemiologically and mathematically well posed. For stable coexistence of the political parties and hence peaceful living, there is need to: keep α_1 as the rate at which individuals are convinced to join the political party P , less than the rates at which individuals leave the party through natural exiting rate and rehabilitation rate; there is need to keep α_2 , as the rate at which individuals are convinced to join the political party Q , less than the rates at which individuals leave the party through natural exiting rate and rehabilitation rate; and keep μ , as the rate at which individuals are convinced to join violent class V , less than the rates at which individuals leave the class through natural exiting rate, violence induced death and rehabilitation. It simply means that the equilibrium is unstable whenever there exists at least one political party and in the presence of violent group. Similarly for stable coexistence of the subsystem (in the absence of violence), α_1 and α_2 need to be checked.

CONCLUSION

A well posed mathematical model exhibiting the dynamics of two political parties in the presence of violence has been proposed and analyzed. Based on the analysis, it has been discovered that our political system in Nigeria is conflated or polluted with violence as a result of relatively high number of individuals of voting or youthful ages joining political parties and violent group rather than how individuals leave the system due to rehabilitation, natural and/or violence-induced death rates. The fact that all the five compartments of the system are interconnected; therefore, increasing the number of partisans, and indirectly increases the number of violent persons as chance of contact and convincing rate are consequently increased.

It is therefore recommended that rehabilitation of violent persons in our societies and indeed other proactive and collaborative measures need to be encouraged or taken by governments (at all levels), political parties and the general societies in order to sanitize our political system for peaceful, socio-political and economic and democratic development in our societies and the country (Nigeria) at large.

COMPETING INTEREST DECLARATION

The authors declare that this work has no financial, personal relationship, or any other reason that could have influenced its conducts.

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