Comparison of some Robust Estimators in Multiple Regression in the Presence of Outliers

Gafar Matanmi Oyeyemi, Ph.D.[1*]; David Adashu Aji, M.Sc.[2]; Adijat Bolanle Ibraheem, Ph.D.[3]; and Akeem Olawunmi Kareem, Ph.D.[4]

[1]Department of Statistics, University of Ilorin, Nigeria.
[2]Department of Mathematics and Statistics, Federal University Wukari, Nigeria.
[3]Department of Physical Sciences, Al-Hikmah University Ilorin, Nigeria.
[4]National Institute for Security Studies, Abuja, Nigeria.

E-mail: gmoyeyemi@gmail.com*
Telephone: +2348068241885

ABSTRACT

Outlier results are one of the problems of Ordinary Least Squares (OLS) in regression analysis. Some estimators have been suggested as alternatives to the Ordinary Least Squares (OLS) estimator to improve the accuracy of the parameter estimates in the linear regression model in the presence of outliers. In this study, six robust estimators of handling the problem of outliers: Robust-M, Robust-MM; Robust-S; Least Trimmed Squares (LTS); Least Median Squares (LMS); and Least Absolute Deviation (LAD) were compared with OLS using Variance criterion. The multiple linear regression model considered, had 4 predictor variables (p = 5) and one dependent variable and there were four levels each of percentage of outliers (10%, 20%, 30%, 40%), variance of outliers (σ_outlier^2=1,50,100,200) and sample sizes (n = 20, 50, 100, 200) were considered through Monte Carlo experiments. The experiment was carried out 1000 times. The results showed that when the variance of outlier is 1, that is, the outliers and variables have standard normal distribution, OLS had the least variance at all sample sizes. But as the variance increases and at all sample sizes, the robust estimators outperformed the OLS. The robust MM had least variance more consistently as the sample size increases at all variance level of the outlier and also as the sample size increases. Therefore, the Robust MM Estimator performed more consistently than the other robust estimators considered.

(Keywords: efficiency, outlier, parameter, rank, robust estimator)

INTRODUCTION

In statistics, an outlier is a data point that differs significantly from other observations. An outlier may be due to variability in the measurement or it may indicate experimental error; the latter are sometimes excluded from the data set (Oyeyemi, et al, 2015). An outlier can cause serious problems in statistical analyses. Outliers can occur by chance in any distribution, but they often indicate either measurement error or that the population has a heavy-tailed distribution. In the former case one wishes to discard them or use statistics that are robust to outliers, while in the latter case they indicate that the distribution has high skewness and that one should be very cautious in using tools or intuitions that assume a normal distribution.

A frequent cause of outliers is a mixture of two distributions, which may be two distinct sub-populations, or may indicate 'correct trial' versus 'measurement error'; this is modeled by a mixture model. Several robust estimators have been proposed to handle the problem of outliers, also some comparisons have been made with the OLS, and therefore, in this research work some common robust estimators (six) were compared with the OLS.

In a study by Bhar (2014), the study looked at the Huber M-estimator as an improvement of the ordinary least squares estimator. In the study, robust M estimator was compared with the ordinary least squares estimator. The study discussed robust regression methods such as; M-estimator, W-estimators, R-estimators, least median of squares estimator, least trimmed of squares estimator, and Reweighted least squares estimator. The most common method of robust regression is M-estimation, introduced by Huber (1973, 1981) that is nearly as efficient as least squares estimator.

AL-Noor and Mohammad (2013) researched on model of robust regression with parametric and non-parametric methods. A simulation study was performed to compare ordinary least squares method; least absolute deviations method; M-Estimators; trimmed least squares estimator and nonparametric regression. In their study, these estimators were compared for vertical outliers, horizontal outliers and both vertical and horizontal outliers based on their mean square error and relative efficiency. The results for the analysis with no contamination showed that ordinary least squares estimator performed better than the other estimators.

Muthukrishnan and Radha (2010) did a study on comparison of robust regression estimators by assessing their coefficient of determination. The study therefore concluded on the note that all robust methods are modification of the traditional methods. Alma (2011) and Badawaire, et al. (2019) researched on comparison of robust regression methods and the study concluded that, S-estimator performed better than the others in terms of efficiencies and both the influence of outliers and leverage points are bounded. Also, the study had shown that MM-estimator breaks down when dealing with high leverage points in small dimensional data.

Yohai (1987) developed the MM-estimator, which is by far the most efficient with a high breakdown point. MM-estimator makes use of other estimators, but for the MM-estimator to possess the high breakdown point property, it was proposed to use the S-estimator as initial estimates to compute the MM-estimator. Yohai's study highlights the properties of the MM-estimator such as; high breakdown point, efficiency, exact fit property and scale equivariance. The robust estimators were compared with the Ordinary least squares using asymptotic biases under contamination. In his study, it was concluded that the MM-estimator was not influenced by outliers as they did to the ordinary least squares.

Alanamu and Oyeyemi (2018) proposed a robust regression method based on regularization of case specific parameter method originally proposed by She and Owen (2011) and Lee, et al. (2012). The study compared the proposed method with other existing methods including OLS using mean square error (MSE) and relative efficiency for various sample sizes and percentages of outliers in both Y and X directions. The study concluded that the proposed robust method performed relatively better than any of the existing methods in terms of efficiency.

METHODOLOGY

The method of ordinary least squares and selected methods of robust regression considered in the study are briefly discussed in the subsequent subsections.

<u>Ordinary Least Squares (OLS)</u>

Given the model $$Y = X\beta + \varepsilon \qquad \textbf{(1)}$$

The OLS aims to minimize:

$$\sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon'\varepsilon = (Y - X\beta)'(Y - X\beta)$$
$$= Y'Y - Y'X\beta - \beta'X'Y + \beta'X'X\beta$$

at the minimum: $\frac{\delta}{\delta\beta}(\sum_{i=1}^{n}\varepsilon_i^2) = 0$

$$= \frac{\partial}{\partial\beta}\left(Y'Y - Y'X\beta - \beta'X'Y + \beta'X'X\beta\right)$$

$$= 0 - X'Y - X'Y + 2X'X\beta$$

$$\Rightarrow 2X'X\beta = 2X'Y$$

OLS estimator $\hat{\beta}$ is obtain by solving the normal equation above yielding:

$$\hat{\beta} = (X'X)^{-1}X'Y \quad \textbf{(2)}$$

Robust M Estimator

Following from M estimation of location, instead of minimizing the sum of squares residuals, a robust regression M estimator minimizes the sum of a less rapidly increasing function of residuals.

$$Min\sum_{i=1}^{n}\rho\left(y_i - \sum_{i=1}^{n}x_{ij}\beta_j\right) = Min\sum_{i=1}^{n}\rho(\varepsilon_i) \quad \textbf{(3)}$$

Robust MM Estimator

MM estimation procedure is to estimate the regression parameter using S estimation which minimizes the scale of the residual from M estimation and then proceed with M estimation. MM estimation aims to obtain estimates that have a high breakdown value and more efficient. Breakdown value is a common measure of the proportion of outliers that can be addressed before these observations affect the model. MM-estimator is the solution of:

$$\sum_{i=1}^{n}\rho_1'(U_i)X_{ij} = 0 \; or \; \sum_{i=1}^{n}\rho_1'\left(\frac{Y_i - \sum_{j=0}^{k}X_{ij}\hat{\beta}_j}{S_{MM}}\right)X_{ij} = 0 \quad \textbf{(4)}$$

Where S_{MM} is the standard deviation obtained from the residual of S estimation and ρ is a Tukey's bi-weight function:

$$\rho(u_i) = \begin{cases} \frac{u_i^2}{2} - \frac{u_i^4}{2c^2} + \frac{u_i^6}{6c^2} & -c \le u_i \le c_i \\ \frac{c^2}{6}, & u_i < -c \; or \; u_i > c \end{cases} \quad \textbf{(5)}$$

Robust S Estimator

The regression estimates associated with M-scales is the S-estimator which was proposed by Rousseeuw and Leroy (2005). S-estimation is based on residual scale of M estimation. The weakness of M estimation is the lack of consideration on the data distribution and not a function of the overall data because it only used the median as the weighted value. This method uses the residual standard deviation to overcome the weaknesses of median. According to Salibian and Yohai (2006), the S-estimator is defined by $\widehat{\beta_s} = min_\beta \, \widehat{\sigma_s}(e_{1,}e_{2,} . . . e_n)$ with determining minimum robust scale estimator $\widehat{\sigma_s}$ and satisfying:

$$min \sum_{i=1}^{n} \rho \left(\frac{y_i - \sum_{i=1}^{n} x_{ij}\beta}{\widehat{\sigma_s}} \right)$$

Least Trimmed Squares (LTS) Estimator

Rousseeuw (1984) developed the least trimmed squares estimator (LTSE) given by:

$$\hat{\beta} = min \sum_{i=1}^{n} (e_i^2) \qquad \textbf{(6)}$$

Where $e_1^2 \leq e_2^2 \leq, \; , \; , \; \leq e_2^n$ are the ordered squared residuals, from smallest to largest. LTSE is computed by minimizing the h ordered squared residuals, where $h = \left(\left[\frac{n}{2} \right] + 1 \right)$, where n and h are the sample size and the trimming constant respectively, (AL-Noor and Mohammad, 2013, Oyeyemi and Odior, 2018).

Least Median of Square (LMS) Estimator

Least squares estimation is a technique used to find parameters for a given equation which provides a best fit for a set of data points. The idea is to minimize squares of the offsets ("the residuals") of the points from the curve. The sum of the squares of the offsets is used instead of the offset absolute values because this allows the residuals to be treated as a continuous differentiable quantity.

$$\min med \; e_i^2$$

Least Absolute Deviation (LAD) Estimator

Chen, et al. (2008) introduced the least absolute deviations (LAD) estimation method which is a robust method in the presence of outliers and asymmetric error terms. The least absolute deviation regression was introduced by Roger Joseph Boscovich in 1757. The LAD aims to get the estimated regression parameters that minimize the sum absolute value of residuals.

$$\hat{\beta}_{LAD} = min|e_i| \qquad \textbf{(7)}$$

Where e_i denotes the i[th] residual.

DATA SIMULATION AND ANALYSIS

The response variable is obtained from the relation given by:

$$y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_4 X_{i4} + \varepsilon_i \, , i = 1, \ldots, n \qquad \textbf{(8)}$$

For the purpose of this study, we simulated four sets of predictors all of size n with sizes 20, 50, 100 and 200 to examine the effect of each of the estimators. Data were generated from the multivariate normal distribution.

The predictors were simulated to be independent and identically distributed random variables as;

$$X_{ij} \sim N(0,1), i = 1,2, \ldots , n; j = 1,2, \ldots , p$$

and the response is simulated with the relationship given below:

$$y = 0.6 + 0.2x_1 + 0.5x_2 + 0.3x_3 + 0.8x_4 + \varepsilon$$

i.e. $\beta' = [\beta_0 = 0.6, \beta_1 = 0.2, \beta_2 = 0.5, \beta_3 = 0.3, \beta_4 = 0.8]$

From the above our number of parameters is 5, implying p = 5.

Ranks were then assigned to these estimators with rank 1 being assigned to the estimator with lowest average value of variance and so on up to the rank 7 to the estimator with the largest average value of variance.

Table 1: Average Variance of Coefficient Estimates when Variance of Outlier is 1 and 50.

Sample Size (n)	Estimator	$\sigma^2_{outlier} = 1$ % of contamination				$\sigma^2_{outlier} = 50$ % of contamination			
		10	20	30	40	10	20	30	40
20	OLS	0.0653	0.0556	0.0607	0.0650	14.6616	29.1446	46.3627	54.3183
	M	0.0696	0.0589	0.0650	0.0701	0.1353	2.3176	6.3008	21.5484
	MM	0.0775	0.0640	0.0665	0.0748	0.0848	0.1106	0.1826	12.8573
	S	0.1838	0.2099	0.1892	0.2020	0.1905	0.1603	0.1568	3.9343
	LTS	0.2528	0.3012	0.2403	0.2646	0.2663	0.2454	0.1894	2.5791
	LMS	0.3849	0.3362	0.3385	0.3517	0.4841	0.4198	0.4489	0.6731
	LAD	0.1029	0.0914	0.0904	0.1040	0.1587	0.5533	0.7208	9.7604
50	OLS	0.0213	0.0231	0.0250	0.0228	6.3661	11.1319	18.5011	21.9465
	M	0.0229	0.0242	0.0261	0.0236	0.0331	0.0672	0.2319	1.4918
	MM	0.0241	0.0239	0.0269	0.0241	0.0260	0.0350	0.0544	0.1185
	S	0.0944	0.0923	0.0868	0.0937	0.0805	0.0888	0.0806	0.0981
	LTS	0.1297	0.1136	0.1178	0.1367	0.1255	0.1216	0.1077	0.0938
	LMS	0.1466	0.1471	0.1400	0.1348	0.1512	0.1385	0.1267	0.1382
	LAD	0.0357	0.0338	0.0381	0.0379	0.0447	0.0617	0.1055	0.1592
100	OLS	0.0105	0.0108	0.0101	0.0104	2.6212	5.0925	6.7889	9.6786
	M	0.0112	0.0114	0.0107	0.0108	0.0160	0.0291	0.0521	0.2300
	MM	0.0110	0.0116	0.0109	0.0107	0.0125	0.0155	0.0177	0.0357
	S	0.0430	0.0465	0.0477	0.0485	0.0444	0.0489	0.0497	0.0693
	LTS	0.0764	0.0760	0.0742	0.0687	0.0719	0.0656	0.0549	0.0559
	LMS	0.0725	0.0712	0.0724	0.0736	0.0635	0.0687	0.0610	0.0730
	LAD	0.0165	0.0166	0.0163	0.0171	0.0201	0.0298	0.0346	0.0530
200	OLS	0.0051	0.0046	0.0054	0.0050	1.2999	2.6750	4.1151	5.0446
	M	0.0054	0.0048	0.0057	0.0054	0.0079	0.0111	0.0263	0.0707
	MM	0.0054	0.0048	0.0057	0.0054	0.0062	0.0069	0.0092	0.0157
	S	0.0269	0.0274	0.0260	0.0306	0.0299	0.0316	0.0376	0.0493
	LTS	0.0416	0.0442	0.0394	0.0462	0.0434	0.0401	0.0382	0.0458
	LMS	0.0371	0.0407	0.0399	0.0406	0.0404	0.0381	0.0428	0.0526
	LAD	0.0077	0.0072	0.0082	0.0081	0.0099	0.0114	0.0176	0.0247

Table 2: Average Variance of Coefficient Estimates when Variance of Outlier is 100 and 200.

Sample Size (n)	Estimator	$\sigma^2_{outlier} = 100$ % of contamination				$\sigma^2_{outlier} = 200$ % of contamination			
		10	20	30	40	10	20	30	40
20	OLS	58.0745	116.7268	185.3102	216.7505	231.3202	467.3444	741.0854	866.0744
	M	0.1365	8.3094	23.8863	85.4444	0.1370	31.7633	94.9561	343.2450
	MM	0.0828	0.0992	0.1595	48.0127	0.0808	0.1022	0.1341	184.8925
	S	0.1824	0.1792	0.1500	13.3692	0.1863	0.1725	0.1496	72.6516
	LTS	0.2572	0.2470	0.1743	13.6091	0.2585	0.2321	0.1773	47.1084
	LMS	0.3795	0.4069	0.3928	0.4354	0.3596	0.3742	0.3819	0.4472
	LAD	0.1587	0.5748	1.0291	36.4419	0.1587	0.5964	2.1319	141.8199
50	OLS	25.5278	44.5282	73.9345	87.7887	102.2982	178.1716	295.6509	351.2031
	M	0.0333	0.0696	0.3099	4.4128	0.0333	0.0709	0.4015	17.1019
	MM	0.0261	0.0337	0.0440	0.1082	0.0261	0.0326	0.0407	0.0682
	S	0.0804	0.0850	0.0811	0.0970	0.0805	0.0839	0.0826	0.0942
	LTS	0.1244	0.1186	0.1013	0.0914	0.1285	0.1241	0.1042	0.0841
	LMS	0.1470	0.1320	0.1264	0.1364	0.1462	0.1351	0.1236	0.1390
	LAD	0.0448	0.0623	0.1145	0.1854	0.0448	0.0630	0.1145	0.1913
100	OLS	10.4966	20.3619	27.1270	38.6961	42.0396	81.4579	108.4725	154.7671
	M	0.0161	0.0294	0.0553	0.3016	0.0161	0.0297	0.0576	0.3875
	MM	0.0123	0.0144	0.0168	0.0300	0.0124	0.0145	0.0164	0.0243
	S	0.04512	0.0478	0.0506	0.0676	0.0452	0.04760	0.0480	0.0667
	LTS	0.0690	0.0656	0.0553	0.0565	0.0690	0.0649	0.0536	0.0571
	LMS	0.0606	0.0670	0.0642	0.0775	0.0591	0.0687	0.0615	0.0773
	LAD	0.0201	0.0298	0.0348	0.0542	0.0201	0.0298	0.0350	0.0545
200	OLS	5.1902	10.7403	16.4254	20.1937	20.7560	43.0537	65.6422	80.8148
	M	0.0080	0.0112	0.0272	0.0852	0.0080	0.0113	0.0280	0.0987
	MM	0.0061	0.0066	0.0080	0.0131	0.0060	0.0064	0.0078	0.0116
	S	0.0291	0.0308	0.0379	0.0505	0.0289	0.0314	0.0383	0.0499
	LTS	0.0432	0.0403	0.0371	0.0456	0.0438	0.0412	0.0370	0.0463
	LMS	0.0408	0.0361	0.0399	0.0521	0.0407	0.0367	0.0397	0.0526
	LAD	0.0099	0.0114	0.0179	0.0250	0.0099	0.0114	0.0180	0.0252

Table 3: Ranks of the Variance of Coefficient Estimates when Variance of Outlier is 1 and 50.

Sample Size (n)	Estimator	$\sigma^2_{outlier} = 1$ % of contamination				$\sigma^2_{outlier} = 50$ % of contamination			
		10	20	30	40	10	20	30	40
20	OLS	1	1	1	1	7	7	7	7
	M	2	2	2	2	2	6	6	6
	MM	3	3	3	3	1	1	2	5
	S	5	5	5	5	4	2	1	3
	LTS	6	6	6	6	5	3	3	2
	LMS	7	7	7	7	6	4	4	1
	LAD	4	4	4	4	3	5	5	4
50	OLS	1	1	1	1	7	7	7	7
	M	2	3	2	2	2	3	6	6
	MM	3	2	3	3	1	1	1	3
	S	5	5	5	5	4	4	2	2
	LTS	6	6	6	7	5	5	4	1
	LMS	7	7	7	6	6	6	5	4
	LAD	4	4	4	4	3	2	3	5
100	OLS	1	1	1	1	7	7	7	7
	M	3	2	2	2	2	2	4	6
	MM	2	3	3	3	1	1	1	1
	S	5	5	5	5	4	4	3	4
	LTS	7	7	7	6	6	5	5	3
	LMS	6	6	6	7	5	6	6	5
	LAD	4	4	4	4	3	3	2	2
200	OLS	1	1	1	1	7	7	7	7
	M	2	2	3	2	2	2	3	6
	MM	3	3	2	3	1	1	1	1
	S	5	5	5	5	4	4	4	4
	LTS	6	7	6	7	6	5	5	3
	LMS	7	6	7	6	5	6	6	5
	LAD	4	4	4	4	3	3	2	2

Table 4: Ranks of the Variance of Coefficient Estimates when Variance of Outlier is 100 and 200.

Sample Size (n)	Estimator	$\sigma^2_{outlier} = 100$ % of contamination				$\sigma^2_{outlier} = 200$ % of contamination			
		10	20	30	40	10	20	30	40
20	OLS	7	7	7	7	7	7	7	7
	M	2	6	6	6	2	6	6	6
	MM	1	1	2	5	1	1	1	5
	S	4	2	1	2	4	2	2	3
	LTS	5	3	3	3	5	3	3	2
	LMS	6	4	4	1	6	4	4	1
	LAD	3	5	5	4	3	5	5	4
50	OLS	7	7	7	7	7	7	7	7
	M	2	3	6	6	2	3	6	6
	MM	1	1	1	3	1	1	1	1
	S	4	4	2	2	4	4	2	3
	LTS	5	5	3	1	5	5	3	2
	LMS	6	6	5	4	6	6	5	4
	LAD	3	2	4	5	3	2	4	5
100	OLS	7	7	7	7	7	7	7	7
	M	2	2	4	6	2	2	5	6
	MM	1	1	1	1	1	1	1	1
	S	4	4	3	4	4	4	3	3
	LTS	6	5	5	3	5	5	4	4
	LMS	5	6	6	5	6	6	6	2
	LAD	3	3	2	2	3	3	2	5
200	OLS	7	7	7	7	7	7	7	7
	M	2	2	3	6	2	2	3	6
	MM	1	1	1	1	1	1	1	1
	S	4	4	5	4	4	4	5	4
	LTS	6	6	4	3	6	5	4	3
	LMS	5	5	6	5	5	6	6	5
	LAD	3	3	2	2	3	3	2	2

DISCUSSIONS OF RESULTS AND CONCLUSION

From Tables 1 to 4, it was discovered that when the variance of the outliers is 1, that is, $\sigma^2_{outlier} \sim N(0,1)$, the OLS estimator outperformed the other estimators since it has least variance because it has least ranks for all levels of sample sizes and percentages of outliers. On the other hand, as the variance of the outliers increased, the Robust MM estimator has least variance because it has least ranks all through the sample sizes and percentage of outliers. Although at smaller sample sizes there was variation in the performance of the Robust MM estimator especially when the variance of the outliers is small but as sample size increases the Robust MM Estimator becomes more consistent.

Therefore, in conclusion, when there is no outlier, the OLS is the most suitable estimator because of its consistency, but when there is outlier(s) the robust estimators outperformed the OLS method. The Robust MM is therefore recommended as the most suitable estimator in fitting multiple linear regression in the presence of outliers.

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