Least Squares Method and Homotopy Perturbation Method for Solving Fractional Integro-differential Equations

T. Oyedepo, M.Sc.[1*]; O.A. Taiwo, Ph.D.[1]; A.F. Adebisi, Ph.D.[2]; C.Y. Ishola, M.Sc.[3]; and O.E. Faniyi, M.Sc.[4]

[1]Department of Mathematics, University of Ilorin, Ilorin, Kwara State, Nigeria.
[2]Department of Mathematical Sciences,Osun state University, Oshogbo, Osun State, Nigeria.
[3]Department of Mathematics, National Open University of Nigeria Jabi, Abuja, Nigeria.
[4]Department of Mathematics, Crown-Hill University, Ilorin, Kwara State, Nigeria.

E-mail: oyedepotaiye@yahoo.com*

ABSTRACT

In this study, two methods for the solution of fractional integro-differential equations (FIDEs) are presented. The fractional derivative is considered in the Caputo sense. The proposed methods are least squares method (LSM) and homotopy perturbation methods (HPM. The proposed method (LSM and HPM) assumed approximate solutions using the Bernstein polynomials as basis functions in each case and then substituted into the general problem considered thereafter reduces the problem to a system of linear algebraic equations and then solved using MAPLE 18. While for homotopy perturbation methods, convex homotopy was constructed involving homotopy parameter p which leads to equating the coefficients of like powers of p in order to obtain the approximate solution. Demonstration of these methods were carried out using some numerical examples. Numerical results validate that the methods are easy to apply and reliable when applied to FIDEs. The graphical solution of each the method is displayed.

(Keywords: Bernstein polynomials, numerical studies, least squares method, homotopy perturbation)

INTRODUCTION

According to Adam (2004), Caputo (1967), Momani and Qaralleh (2006), and Podlubny (1999), fractional calculus has a long record dating back to 1695 when the derivative of order $\propto = \frac{1}{2}$ had been reported (Adam, 2004). The speculation of derivatives and integrals of non-integer order goes back to Leibniz, Liouville, Gr¨unwald, Letnikov and Riemann. There are many fascinating or exciting books about fractional calculus and fractional differential equations (Caputo,1967; Munkhammar, 2005; and Podlubny, 1999).

The help of fractional differentiation for the mathematical modeling of real-world physical problems has been extensively increasing in recent years (e.g., the modeling of earthquakes, reducing the spread of viruses, control of the memory behavior of electric socket, etc.). Understanding of definitions and benefit of fractional calculus will be made more understandable by quickly discussing some compulsory but relatively simple mathematical definitions that will appear in the study of these concepts. These are the Gamma Function, the Beta Function, and Riemann-Liouville Fractional Integral. The help of this class of calculus for solving problems in the branch of physics, chemistry, engineering, and technology has been extensive spread in recent years by many researchers.

Podlubny (1999) suggested a method for the solution of FIDEs called using Adomian decomposition method (ADM). Rawashdeh (2006) also employed collocation method for solving fractional integro-differential equations. Mittal and Nigam (2008) suggested a method for the solution of fractional integro-differential equations called Adomian Decomposition Method. ADM involves the construction of Adomian polynomials which are somehow strenuous to get. Hashim et al. (2009) employed Homotopy Perturbation and Homotopy Analysis methods for solving FIDEs. A method called the application of the fractional differential transform by Nazari and Shahmorad (2010) was employed for solving fractional-order integro-differential equations with nonlocal boundary conditions. He (1999) also gives some application of nonlinear fractional differential equations and their approximation.

Taiwo and Odetunde (2011) applied Iterative Decomposition Method (IDM) for finding numerical approximation of fractional Integro-differential equations. Abdollahpoor (2014) employed least square method for treating nonlinear fourth order integro-differential equations. Applied a method called an efficient method with the help of Bernstine polynomials for solving fractional differential equations.

Mohammed (2014) employed least squares method for solving fractional integro-differential equations using shifted Chebyshev polynomial of the first kind as basis function. In this work, using the ideas of Mohammed (2014) and Oyedepo, (2016), we proposed an alternative method, called least square and Homotopy perturbation methods by Bernstein polynomials as basis functions. The general forms of the class of problem considered in this work is given as:

$$D^{\alpha}u(x) = f(x) + \int_0^1 k(x,t)u(x)dt, \quad o \le x,t \le 1, \tag{1}$$

With the following supplementary conditions:

$$u^{(i)}(0) = \delta_{i,} \quad i = 0,1,2,\dots,n-1, \quad n-1 < \alpha \le n, n \in, N \tag{2}$$

Where $D^{\alpha}u(x)$ indicates the $\propto th$ Caputo fractional derivative of $u(x)$, $f(x)$, $K(x,t)$ are given smooth functions, x and t are real variables varying [0,1] and $u(x)$ is the unknown function to be determined.

Some relevant basic definitions:

Definition 1: Fraction Calculus involves differentiation and integration of arbitrary order (all real numbers and complex values). Example $D^{\frac{1}{2}}, D^{\pi}, D^{2+i}$ e.t.c

Definition 2: Gamma function is defined as:

$$\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt \tag{3}$$

This integral converges when real part of z is positive $(Re(z) \le 0)$.

$$\Gamma(1+z) = z\Gamma(z) \tag{4}$$

When z is a positive integer:

$$\Gamma(z) = (z-1)\,! \tag{5}$$

Definition 3: Beta function is defined as:

$$B(v,m) = \int_0^1 (1-u)^{v-1}u^{m-1}du = \frac{\Gamma(v)\Gamma(m)}{\Gamma(v+m)} = B(v,m), \text{ where } v,m \in R_+ \tag{6}$$

Definition 4: Riemann – Liouville fractional integral is defined as:

$$J^{\alpha}f(x) = \frac{1}{\Gamma(x)}\int_0^x \frac{f(x)}{(x-t)^{1-\alpha}}dt, \quad \alpha > 0, x > 0, \tag{7}$$

J^{α} denotes the fractional integral of order $\propto$

Definition 5: Riemann – Liouville fractional derivative denoted $D^{\propto}$is defined as:

$$D^{\propto}J^{\alpha}f(x) = f(x) \tag{8}$$

Definition 6: Riemann - Liouville fractional derivative defined as:

$$D^{\alpha}f(x) = \frac{1}{\Gamma(\mathrm{m}-\propto)}\int_0^x (x-s)^{m-\propto-1} f^{m}(s)ds, \tag{9}$$

m is positive integer with the property that $m-1 <\propto< m$.

Definition 7: The Caputor Factional Derivative is defined as:

$$D^{\alpha}f(x) = \frac{1}{\Gamma(\mathrm{m}-\propto)}\int_0^x (x-s)^{m-\propto-1} f^{m}(s)ds \tag{10}$$

Where m is a positive integer with the property that $m-1 <\propto< m$

For example if $0 <\propto< 1$ the caputo fractional derivative is:

$$D^{\alpha}f(x) = \frac{1}{\Gamma(1-\propto)}\int_0^x (x-s)^{-\propto} f^{1}(s)ds \tag{11}$$

Hence, we have the following properties:

(1) $J^{\alpha}\, J^{v} f = j^{\alpha+v} f, \alpha, v > 0, f \in C_{\mu}, \mu > 0$

(2) $J^{\alpha} x^{\gamma} = \frac{\Gamma(\lambda+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma,}\ \alpha > 0, \gamma > -1, x > 0$

(3) $J^{\alpha}\, D^{\alpha} f(x) = f(x) - \sum_{k=0}^{m-1} f^{k}(0^{+})\frac{x^k}{k}, \qquad x > 0, m-1 < \alpha \leq m$

(4) $D^{\alpha}\, J^{\alpha} f(x) = f(x), \quad x > 0, m-1 < \alpha \leq m,$

(5) $D^{\alpha} C = 0, C$ is the constant,

(6) $\begin{cases} 0, & \beta \in N_0, \beta < [\alpha], \\ D^{\alpha} x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha}, & \beta \in N_0, \beta \geq [\alpha], \end{cases}$

Where $[\alpha]$ denoted the smallest integer greater than or equal to α and $N_0 = \{0,1.2, \ldots\}$

Definition 8: Bernstein basis polynomials: A Bernstein polynomial [8] of degree n is defined by:

$$B_{i,n}(x) = \binom{n}{i} x^{i}(1-x)^{n-i} \quad i = 0, 1 \ldots n, \tag{12}$$

where,

$$\binom{n}{i} = \frac{n!}{i!(n-1)!} \tag{13}$$

Often, for mathematical convenience, we set $B_{i,n}(x) = 0$ if < 0 or $i > n$.

Definition 9: Bernstein polynomials: A linear combination Bernstein basis polynomials:

$$u_n(x) = \sum_i^n a_i B_{i,n}(x) \tag{14}$$

is the Bernstein polynomial of degree n where $a_{i,}$ $i = 0,1,2, \ldots \ldots$ are constants.

Examples:

The first few Bernstein basis polynomials are:

$$u_0(x) = 1, u_1(x) = a_0(1-x) + a_1 x,\ u_2(x) = a_0(1-2x+x^2) + a_1(2x-2x^2) + a_2 x^2 \tag{15}$$

Definition 10: In this work, we defined absolute error as:

Absolute Error $=|Y(x) - y_n(x)|;\quad 0 \le x \le 1$ (16)

MATERIALS AND METHOD

Demonstration of the Proposed Methods

In this section, we demonstrated the two proposed methods mentioned above.

Least Squares Method (SLM)

The least squares method is employed applied to (1) with Bernstein polynomials as basis functions. To approximate the unknown function $u(x)$ we assume an approximation solution of the form (14). Consider equation (1) operating with $J^{\propto}$ on both sides as follows:

$$J^{\propto} D^{\alpha} u(x) = J^{\propto} f(x) + J^{\propto}(\int_0^1 k(x,t) u(t) dt) \tag{17}$$

$$u(x) = \sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^1 k(x,t) u(t) dt] \tag{18}$$

Substituting (14) into (17) yields:

$$\sum_i^n a_i B_{i,n}(x) = \sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^1 k(x,t) \sum_i^n a_i B_{i,n}(t)\, dt] \tag{19}$$

Hence, we obtained the residual equation as:

$$R(a_{0,} a_1, \dots \dots \dots, a_{n)} = \sum_i^n a_i B_{i,n}(x) - \{\sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^1 k(x,t) \sum_i^n a_i B_{i,n}(t)\, dt]\} \tag{20}$$

Let

$$S(a_{0,} a_1, \dots \dots \dots, a_n) = \int_0^1 [R(a_{0,} a_1, \dots \dots \dots, a_n)]^2 w(x) dx \tag{21}$$

Where $w(x)$ is the positive weight function defined in the interval, [a, b]. In this work, we take $w(x) = 1$ for simplicity.

$$S(a_{0,} a_1, \dots \dots \dots, a_n) = \int_0^1 \left\{\sum_i^n a_i B_{i,n}(x) - \{\sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + [\int_0^1 k(x,t) \sum_i^n a_i B_{i,n}(t)\, dt]\}\right\}^2 \tag{22}$$

In order to minimize Equation (22), the values of $a_i\ (i \ge 0)$ is obtained by finding the minimum value of S as:

$$\frac{\partial S}{\partial a_i} = 0, i = 0.1, \dots \dots \dots, n \tag{23}$$

Applying (23) to (22) for various values of $a_i (i \ge 0)$. Thus, (22) is then simplified in other to obtain $(\mathrm{n}+1)$ linear algebraic system of equations in $(\mathrm{n}+1)$ unknown constants a'_i s, which is then solved using

Gaussian elimination method or maple 18 to obtain the unknown constants $a_i (i = 0(1)n)$, which are then substituted back into the assumed approximate solution to give the required approximation solution.

HOMOTOPY PERTURBATION METHOD

Consider equation (1.1) operating with $J^{\propto}$ on both sides as follows:

$$J^{\propto} D^{\alpha} u(x) = J^{\propto} f(x) + J^{\propto}(\int_0^1 k(x,t)u(t)dt) \tag{24}$$

$$u(x) = \sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^1 k(x,t)u(t)dt] \tag{25}$$

We solved (25) by the homotopy perturbation method, we constructed the following convex homotopy:

$$u(x) = \sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + P[J^{\propto} \int_0^1 k(x,t)u(t)dt] \tag{26}$$

$$u(x) = \sum_i^n P^i u_i(x) \tag{27}$$

Expanding (27) further to have:

$$u(x) = P^0 u_0(x) + P^1 u_1(x) + P^2 u_2(x) + \ \ldots \tag{28}$$

Substituting (28) into (26) yields:

$$u_0(x) + P^1 u_1(x) + P^2 u_2(x) + \ \ldots = \sum_{k=0}^{m-1} u^k(0^+)\frac{x^k}{k!} + J^{\propto} f(x) + P[J^{\propto} \int_0^1 k(x,t)u(t)dt] \tag{29}$$

Equating the coefficients of the like powers of P, to get the following set of fractional differential equations, in order to determine the components $u_0(x), u_1(x)$, $u_2(x)$, ...:

$$P^0 u_0(x) = J^{\propto} f(x) \tag{30}$$

$$P^1 u_1(x) = J^{\propto}(\int_0^1 k(x,t)(P^0 u_0(t)dt) \tag{31}$$

$$P^2 u_2(x) = J^{\propto}(\int_0^1 k(x,t)(P^1 u_1(t)dt)$$

.

$$P^n u_n(x) = J^{\propto}(\int_0^1 k(x,t)(P^{n-1} u_{n-1}(t)dt) \tag{32}$$

Therefore, combining all the determined components $u_0(x), u_1(x)$, $u_2(x)$, ...: gives the required approximation solution as

$$u(x) \ = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots \tag{32}$$

Numerical Examples: In this section, we applied above two methods on some examples.

Example 1: Consider the following fractional Integro-differential equation:

$$D^{1/2} u(x) = \frac{(8/3)x^{3/2} - 2x^{1/2}}{\sqrt{\pi}} + \frac{x}{12} + \int_0^1 xtu(t)dt, \ 0 \le x \le 1, \tag{33}$$

Subject to $u(0) = 0$, with exact solution $U(x) = x^2 - x$ (34)

Tables of Results

Table 1: Numerical Results of Example 1.

X	Exact Solution	Approximate Solution of standard least squares method (LSM) n=3	Approximate solution of perturbed (HPM) n=3	Absolute error of standard least squares method (LSM) n=3	Absolute error of homotopy perturbation method (HPM) n=3
0.0	0.00	-0.0000001112	0.00000000000	1.11228E-7	0.000000
0.1	-0.09	-0.0899813572	-0.08996170804	1.86427E-5	3.8291E-5
0.2	-0.16	-0.1599663500	-0.15991071810	3.33248E-5	8.9281E-5
0.3	-0.21	-0.2099551127	-0.20985286610	4.48873E-5	1.4713E-4
0.4	-0.24	-0.2399476691	-0.23979023500	5.23309E-5	2.0976E-4
0.5	-0.25	-0.2499440422	-0.24972402790	5.59578E-5	2.7597E-4
0.6	-0.24	-0.2399442554	-0.23965505730	5.57446E-5	3.4494E-4
0.7	-0.25	-0.2099483324	-0.20958391940	5.16676E-5	4.1608E-4
0.8	-0.16	-0.1599562961	-0.15951107590	4.37039E-5	4.8892E-4
0.9	-0.09	-0.0899681702	-0.08943689878	3.18298E-5	5.6310E-4
1.0	0.00	0.00160222E-2	0.000638304940	1.18146E-5	6.3830E-4

Table 2: Numerical Results of Example 1.

X	Exact Solution	Approximate Solution of standard least squares method (LSM) n=10	Approximate solution of perturbed (HPM)n=10
0.0	0.00	0.00000000000	0.0000000
0.1	-0.09	-0.0899813835	1.8616E-5
0.2	-0.16	-0.1599663689	3.3631E-5
0.3	-0.21	-0.2099551031	4.4896E-5
0.4	-0.24	-0.2399476390	5.2361E-5
0.5	-0.25	-0.2499440069	5.5993E-5
0.6	-0.24	-0.2399442273	5.5772E-5
0.7	-0.25	-0.2099483152	5.1684E-5
0.8	-0.16	-0.1599562823	4.3717E-5
0.9	-0.09	-0.0899681754	3.1824E-5
1.0	0.00	0.0001611019	1.6110E-5

Examples 2: Consider the following fractional Integro-differential equation:

$$D^{\frac{5}{6}}u(x) = -\frac{3}{91}\frac{x^{\frac{1}{6}}\Gamma 5/6(-91+216x^2)}{\pi} + (5-2e)x + \int_0^1 xe^t u(t)dt, \ \leq x \leq 1, \tag{35}$$

Subject to $u(0) = 0$ with the exact equation $U(x) = x - x^3$ (36)

Table 3: Numerical Results of Example 2

X	Exact Solution	Approximate solution of standard least squares method (LSM) n=3	Approximate solution of perturbed least squares method (HPM) n=3	Absolute error of standard least squares method(LSM) n=3	Absolute error perturbed least squares method (HPM) n=3
0.0	0.00	0.0000003.826	0.00000000000	3.82691E-7	0.000000
0.1	0.099	0.0989574148	0.09864820214	4.23906E-5	3.5179E-4
0.2	0.192	0.1919131983	0.19081102930	8.68017E-5	1.1889E-3
0.3	0.273	0.2728702735	0.27055223920	1.29726E-4	2.4477E-3
0.4	0.336	0.3358311807	0.33190308820	1.68819E-4	4.0969E-3
0.5	0.375	0.3747984600	0.36888491470	2.01540E-4	6.1150E-3
0.6	0.384	0.3837746522	0.37551409340	2.25347E-4	8.4859E-3
0.7	0.357	0.3567622973	0.34580402570	2.37702E-4	1.1195E-2
0.8	0.288	0.2877639356	0.27376613910	2.36064E-4	1.4233E-2
0.9	0.171	0.1707821077	0.15341046120	2.17892E-4	1.7589E-2
1.0	0.00	-1.806464E-4	-0.0212540233	1.80646E-4	2.1254E-2

Table 4: Numerical Results of Example 2.

X	Exact Solution	Approximate solution of standard least squares method (HPM) n=10	Error of homotopy perturbation method (HPM) n=10
0.0	0.000	0.0000000000	0.0000000
0.1	0.099	0.0989562985	4.3701E-5
0.2	0.192	0.1919089615	9.1038E-5
0.3	0.273	0.2728679364	2.7286E-4
0.4	0.336	0.3358156739	1.8432E-4
0.5	0.375	0.3747751442	2.2485E-4
0.6	0.384	0.3837421607	2.5783E-4
0.7	0.357	0.3567192739	2.8072E-4
0.8	0.288	0.2877090093	2.9099E-4
0.9	0.171	0.1707138756	2.8612E-4
1.0	0.000	0.0002636310	2.6363E-4

Graphical Representation of the Two Methods

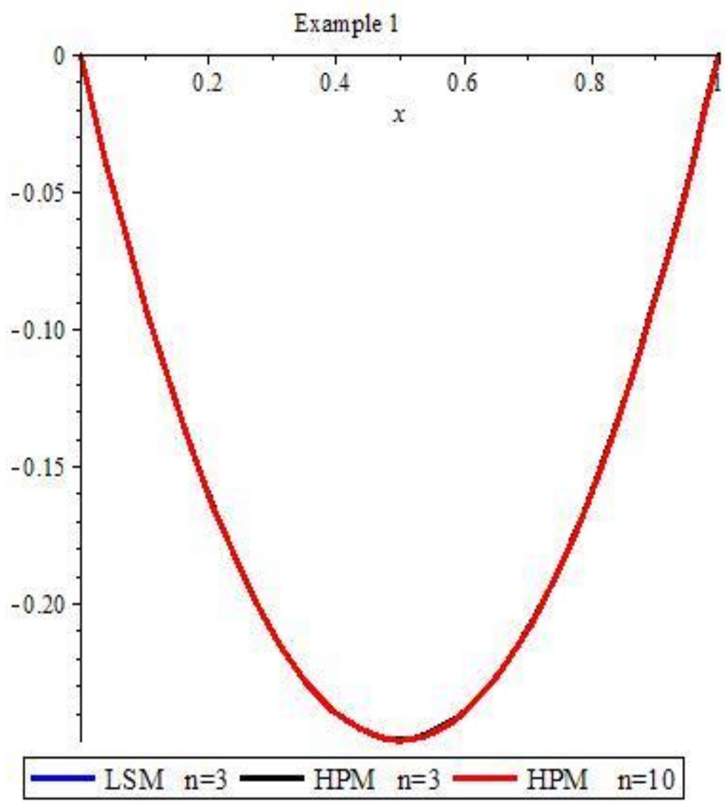

Figure 1: Numerical Results of Example 1.

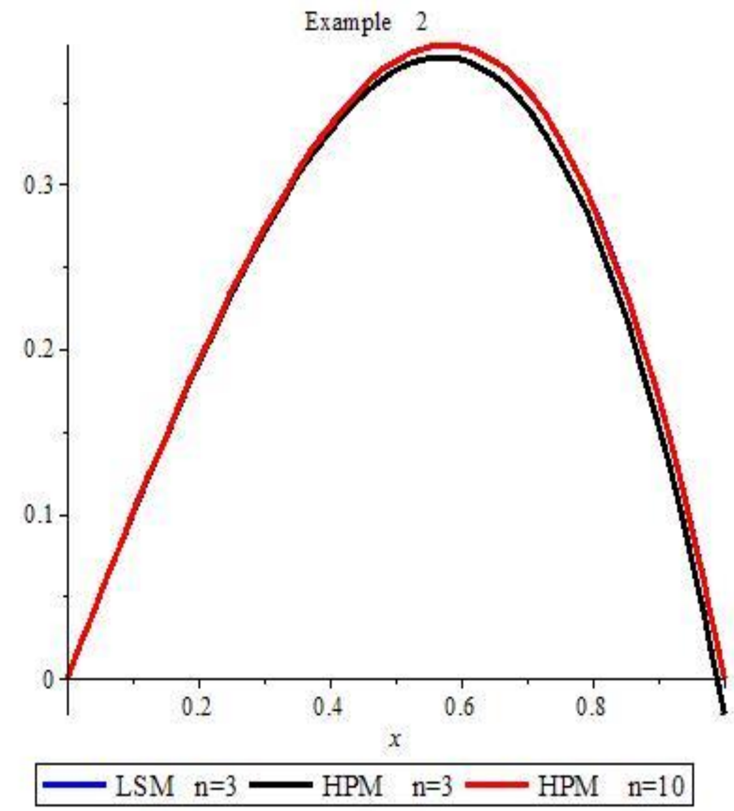

Figure 2: Numerical Results of Example 2.

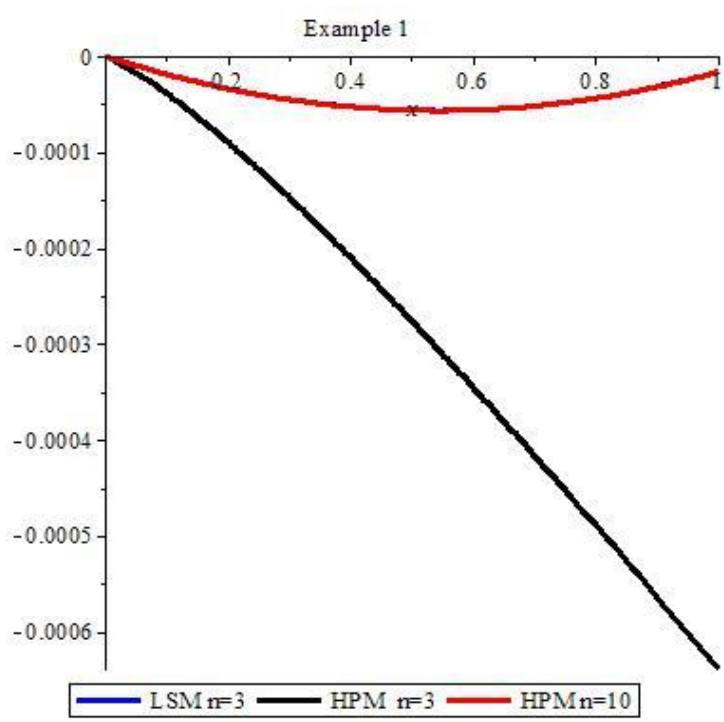

Figure 3: Error of Example 1.

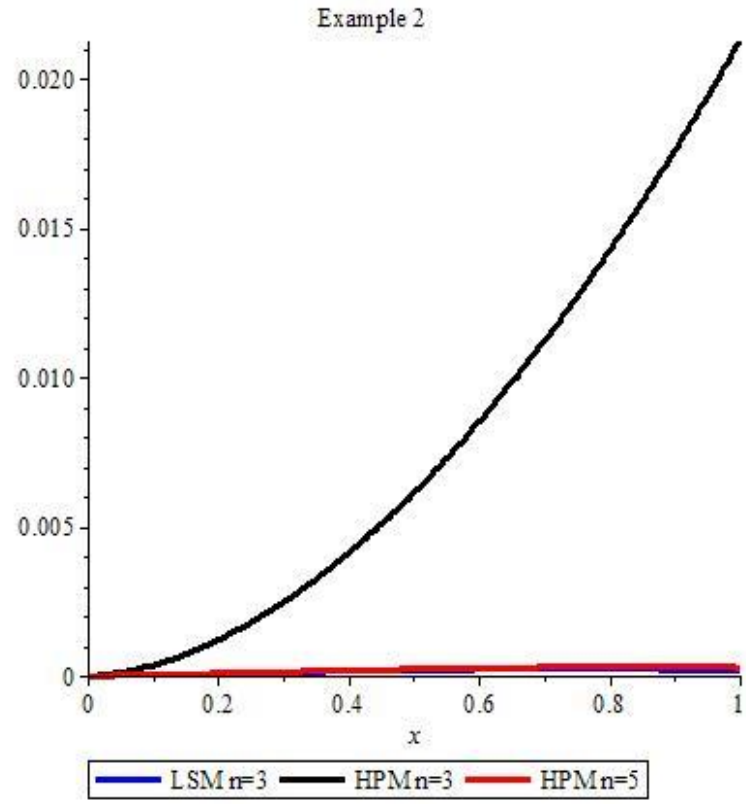

Figure 4: Error of Example 2.

CONCLUSION

The study showed that the methods were successfully used for solving fractional integro differential equations. We observed that the LSM involves less computational effort when compared with HPM. The results obtained showed that the two methods and compared with the exact solutions shown that there is a similarity between the exact and the approximate solution. Calculations showed that LSM and HPM are powerful and efficient techniques in finding very good solutions for this type of equation. The results were presented in graphical forms to further demonstrate the relationship between the methods.

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ABOUT THE AUTHORS

T. Oyedepo, is a Ph.D. student in the Department of Mathematics, University of Ilorin, Nigeria. He holds a Master of Science in Mathematics from University of Ilorin, Nigeria. His research interests are in numerical analysis.

O.A. Taiwo, is a Professor of Mathematics at the Department of Mathematics, University of Ilorin, Nigeria. He holds Ph.D., M.Sc., and B.Sc. degrees from the University of Ilorin, Ilorin, Nigeria. His research interests are in numerical analysis.

A.F. Adebisi, is a Lecturer in the Department of Mathematical Sciences, Osun State University Oshogbo, Nigeria. She holds Ph.D., M.Sc., and B.Sc. degrees from the University of Ilorin, Ilorin, Nigeria. Her research interests are in numerical analysis and mathematical modelling,

C.Y. Ishola, is a Lecturer in the Department of Mathematics, National Open University of Nigeria Jabi, Abuja, Nigeria. She holds M.Sc. and B.Sc. degrees from the University of Ilorin, Ilorin, Nigeria. Her research interests are in numerical analysis.

O.E. Faniyi, is a Lecturer in the Department of Mathematics at Crown-Hill University, Ilorin, Nigeria. He holds M.Sc. and B.Sc. degrees from the University of Ilorin, Ilorin, Nigeria. His research interests are in numerical analysis.

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