Solution of Cholera Disease Model by Parameter Expansion Method

A.A. Ayoade, Ph.D.[1]; O.J. Peter, M.Tech [2*]; S. Amadiegwu, M.Sc.[3]; A.I. Abioye, M.Sc.[3]; A.A. Victor, M.Sc.[2]; and A.F. Adebisi, Ph.D.[4]

[1]Department of Mathematics, Kola Daisi University, Ibadan, Nigeria.
[2]Department of Mathematics, University of Ilorin, Ilorin, Nigeria.
[3]Department of Mathematics, School of General Studies, Maritime Academy of Nigeria, Oron, Akwa Ibom State. Nigeria.
[4]Department of Mathematical Sciences, Osun State University, Oshogbo, Osun State, Nigeria.

E-mail: peterjames4real@gmail.com

ABSTRACT

Nonlinear problems are difficult to solve analytically yet their solutions can be obtained by a number of semi-numerical-analytical methods. To that end, we propose a deterministic cholera model and use parameter expansion method to attempt the solution of the problem analytically. The problem is solved and the solutions obtained confirm the efficiency of parameter expansion method in solving nonlinear infectious disease models.

(Keywords: nonlinear problems, analytic methods, cholera model, parameter expansion method)

INTRODUCTION

Cholera is an infection of the small intestine characterized by profuse watery diarrhea, vomiting, circulatory collapse, and shock (CDC, 2005). Cholera also causes rapid dehydration and electrolyte imbalance and can lead to death if prompt treatment is not given. Such symptoms are usually manifested in a period ranging between 18 hours and 5 days of incubation potentially in conjunction with an important liquid loss that may be close to one liter per hour (Sacks, 2004). The treatment basically consists of rehydration therapy which may be complemented with appropriate antibiotic assistance depending on whether complication occurs or not.

Despite implementation of various intervention strategies towards eradication of cholera disease, the disease still remains a major health concern in Third World economies. Available reports show that the global outbreak of cholera is rising in Africa and all less developed countries with a strong trend of cholera outbreaks in Haiti (2010-2011), Cameroon (2010-2011), Kenya (2010), Vietnam (2009), Zimbabwe (2008-2009), Iraq (2008), the Democratic Republic of Congo (2008) and India (2007) (Al-Arydah et al., 2013). The World Health Organistion (WHO) reported 245,933 cholera cases and 3,034 death cases across 48 countries in which 67% cases occurred in Africa (WHO, 2013).

Codeco (2001) was the first to identify the impact of environmental component on cholera transmission. The author built a cholera model that incorporated the concentration of *V. cholerae* in the water supply denoted by B, into a regular SIR system to form a combined environment-to-human $(SIR-B)$ epidemiological model. This model enhances a careful analysis of the complex interaction between human host and environmental pathogen towards a better understanding of the cholera transmission mechanism and as a result, it has motivated the development of several other cholera models.

Building on Codeco (2001), Ochoche (2013) and Fatima et al. (2014) proposed a mathematical model for the control of cholera transmission dynamics using water tratment and environmental sanitation as control measures. Their results show that water treatment and environmental sanitation are effective methods of controlling cholera disease. Al-Arydah et al. (2013) included education to water chlorination in the control of cholera in their model and found that education is more effective than chlorination in limiting bacteria and the number of cholera cases.

Also, Edward and Nyerere (2015) developed a model extended from Codeco (2001) to study the mode of spread and control of cholera using therapeutic treatment and all forms of sanitary measures. Their results suggest that in any outbreak of cholera, there is need to quarantine infected people and quickly treat them with therapeutic measures (such as rehydration and antibiotics). In addition, applying all possible measures to prevent ingestion of *Vibrio cholerae* through water, food and the entire environment is mandatory.

Furthermore, Wang and Modnak (2011) developed a model adapted from Mukandavire et al. (2011) to study the effects of different control measures (vaccination, therapeutic treatment and water sanitation) on cholera epidemics. They discovered that a combination of multiple intervention methods generally achieve better results than a single control such as vaccination only.

Cholera vaccine was invented in 1896 by a German scientist Willhelm Kolle. It was added to the WHO recommendation for cholera prevention and control in 2010 (Trawick, 2017). Concerns about vaccines feasibility, timeliness, and acceptability by the communities, as well as fear of diverting resources to other medical programmes are some of the factors responsible for late recommendation of vaccines by the WHO for cholera management (Li, 2017).

On the use of vaccination, Azman and Lessler (2014) constructed metapopulation models of a cholera-like disease and compared strategies for allocating vaccines across multiple areas with heterogeneous transmission efficacy. Their results correspond to the results obtained in Chao et al. (2011) and show that connectivity between populations, transmission efficacy, vaccination timing and the amount of vaccine available all shape the performance of different strategies though targeting vaccination in transmission hotspots (i.e. areas with high transmission efficiency) might be a potential approach to efficiently allocate vaccine.

The process of solving nonlinear problems is tasking and rigorous, yet obtaining the solutions of nonlinear problems becomes imperative because most real life phenomena are nonlinear and are described by nonlinear equations. Nonlinear equations can be solved by numerous methods (e.g Homotopy perturbation method, Homotopy analysis method, Lyapunov artificial small parameter method, δ-expansion method, Adomian decomposition method, Variational iteration method, etc.). Parameter expansion method (PEM) is also a powerful tool for obtaining approximate analytic series solutions to nonlinear problems. The basic feature of PEM is to expand the solution and some parameters in the equation (He, 2006). The parameter expansion method is promising because it is straightforward to handle nonlinear problems. Besides, the method is easy to understand with a basic knowledge of advanced calculus (Sweilam and Khader, 2010).

Parameter expansion method was introduced by He (2006). The method has been applied successfully to various physics and engineering problems (Nofal et al., 2013; Swilam and Al-Bar, 2009; Sweilam and Khader, 2010; Peter and Awoniran, 2018). Apart from physics and engineering disciplines, parameter expansion method is a powerful tool in Mathematical Biology and was used by Oguntolu et al. (2015) to obtain the solution of an infectious disease model adapted from Codeco (2001). In what follows, we shall apply parameter expansion method to solve a cholera model where the procedure is in Sweilam and Khader (2010).

MATERIALS AND METHODS

A cholera transmission model was designed by Fung (2014) by assuming that, an individual can only be infected if he consumed contaminated water. The model also assumed that the human population size at a particular time was categorized into subclasses S(t), I(t), and R(t), respectively, while the pathogen population was denoted by B(t). The human population was recruited at a rate μ_b, and the recovery rate for human population was given by γ. The force of infection was λ which was a function of β, the rate of exposure of susceptible individuals to contaminated water, $\aleph$ was the pathogen population that yielded 50% chance of catching cholera on consuming dirty water. ε was the rate at which infectious individuals contributed to the growth of pathogen while δ was the rate at which pathogen were removed from the aquatic environment naturally. Natural death and cholera induced death rates for the host populations were represented by μ_c and μ_d respectively. N was the total human population. The model by Fung (2014) is described by the following system of ODEs

$$
\left.\begin{aligned}
\frac{dS}{dt} &= -\lambda S + \mu_b N - \mu_d S \\
\frac{dI}{dt} &= \lambda S - \gamma I - (\mu_c + \mu_d) I \\
\frac{dR}{dt} &= \gamma I - \mu_d R \\
\frac{dB}{dt} &= \varepsilon I - \delta B.
\end{aligned}\right\} \qquad \textbf{(1)}
$$

$$\lambda = \beta\left[\frac{B}{(B+\aleph)}\right] \text{and } N = S + I + R$$

where

$S(t) = \textit{susceptible individuals at time } t$

$R(t) = \textit{recovered individuals at time } t$

$I(t) = \textit{infected individuals at time } t$

$N(t) = \textit{total human population at time } t$

$B(t) = \textit{concentration of } V.\textit{cholerae in the water reservoir or supply at time } t$

$\mu_c = \textit{death rate due to disease}$

$\mu_d = \textit{death rate unrelated to disease}$

δ = *rate of removal of pathogen unrelated to water treatment*

$\mu_b = \textit{birth rate}$

$\lambda = \textit{force of infection}$

$\beta = \textit{"contact rate" between the susceptible population and contaminated water}$

$\aleph = \textit{pathogen concentration that yields } 50\% \textit{ chance of catching cholera}$

$\gamma = \textit{recovery rate of infected individuals } \left(\frac{1}{\gamma} = \textit{duration of infectiousness}\right)$

One of the novel results in Fung (2014) is that cholera is bound to explode from time to time in societies where poverty and ignorance are the order of the day. However, the study in Fung (2014) did not consider disease control measures that played vital roles in cholera dynamics but assumed permanent immunity after recovery which was not a realistic assumption. All these shortcomings necessitated the new model. To come about the new model, we incorporate the control parameters θ, ρ and ω and the possibility of disease transmission from man-to-man denoted by β_2 into the ODE model in Fung (2014) in equation (1). The new system is described by the following set of equations:

$$\left.\begin{aligned}\frac{dS}{dt} &= \pi - \mu S - (1-\theta)\frac{\beta_1 BS}{(B+\aleph)} - (1-\theta)\beta_2 IS + \sigma R \\ \frac{dI}{dt} &= (1-\theta)\frac{\beta_1 BS}{(B+\aleph)} + (1-\theta)\beta_2 IS - (\mu + \mu_c + \rho)I \\ \frac{dR}{dt} &= \rho I - \mu R - \sigma R \\ \frac{dB}{dt} &= (1-\theta)\varepsilon I - (\delta + \omega)B\end{aligned}\right\} \quad \textbf{(2)}$$

with initial conditions $S(0) = S_0,\ I(0) = I_0,\ R(0) = R_0,\ B(0) = B_0$

All the parameters and state variables in equation (1) that appear in equation (2) remain as defined in equation (1). However, in equation (2), π is the recruitment rate into susceptibility, θ is the rate of awareness (i.e education), β_1 and β_2 are contact rates between susceptible individuals and contaminated water and, susceptible individuals and infectious individuals respectively, μ is natural human mortality rate,σ is the rate of losing immunity while ρ and ω are rate of treatment of infectious individuals and death rate of pathogen due to water sanitation, respectively.

The new model is built around the following assumptions:

(1) The rate of application of the control parameters is high enough to produce desirable results.

(2) The flow between the compartments is continuous. That is, there is no permanent immunity.

METHOD OF SOLUTION

Based on the assumptions on which the model is built and the results in Wang and Modnak (2011), the disease was checked in the population. We do not need to investigate the effect of the controls on the dynamics of the disease since their rates of application are assumed enough to produce desirable results. We shall only attempt the analytical solution of the system via parameter expansion method.

Let $S = v,\ I = x,\ R = y,\ B = z$

Also let

$$\begin{aligned} k_1 &= (1-\theta)\beta_1 \\ k_2 &= (1-\theta)\beta_2 \\ k_3 &= (1-\theta)\varepsilon \\ k_4 &= (\mu + \mu_c + \rho) \\ k_5 &= (\mu + \sigma) \\ k_6 &= (\delta + \omega) \end{aligned}$$

Then the system of equations (2) becomes:

$$\left.\begin{aligned}\frac{dv}{dt} &= \pi - \mu v - \frac{k_1 zv}{(z+\aleph)} - k_2 xv + \sigma y \\ \frac{dx}{dt} &= \frac{k_1 zv}{(z+\aleph)} + k_2 xv - k_4 x \\ \frac{dy}{dt} &= \rho x - k_5 y \\ \frac{dz}{dt} &= k_3 x - k_6 z\end{aligned}\right\} \quad \textbf{(3)}$$

with initial conditions

$v(0) = v_0,\ x(0) = x_0,\ y(0) = y_0,\ z(0) = z_0$

According to PEM, the solutions of the unknown and coefficients $1,\ \pi,\ k_1,\cdots,k_6$ and ρ are expanded into a series of an artificial parameter, say p, respectively in the form:

$$\left.\begin{array}{ll} v(t) & = v_o(t) + pv_1(t) + p^2 v_2(t) + h.o.t \\ x(t) & = x_o(t) + px_1(t) + p^2 x_2(t) + h.o.t \\ y(t) & = y_o(t) + py_1(t) + p^2 y_2(t) + h.o.t \\ z(t) & = z_o(t) + pz_1(t) + p^2 z_2(t) + h.o.t \\ 1 & = r^2 + pr_1 + p^2 r_2 + h.o.t \\ (1-\theta) & = s^2 + ps_1 + p^2 s_2 + h.o.t \\ \mu & = pa_1 + p^2 a_2 + h.o.t \\ \pi & = pb_1 + p^2 b_2 + h.o.t \\ \delta & = pc_1 + p^2 c_2 + h.o.t \\ \omega & = pd_1 + p^2 d_2 + h.o.t \\ \rho & = pe_1 + p^2 e_2 + h.o.t \\ \sigma & = pf_1 + p^2 f_2 + h.o.t \\ \varepsilon & = pg_1 + p^2 g_2 + h.o.t \\ \aleph & = pq_1 + p^2 q_2 + h.o.t \end{array}\right\} \tag{4}$$

such that $k_1,\ k_2,\ k_3,\ k_4,\ k_5,\ k_6$ are given as:

$$\left.\begin{array}{ll} k_1 & = ph_1 + p^2 h_2 + h.o.t \\ k_2 & = pi_1 + p^2 i_2 + h.o.t \\ k_3 & = pj_1 + p^2 j_2 + h.o.t \\ k_4 & = pl_1 + p^2 l_2 + h.o.t \\ k_5 & = pm_1 + p^2 m_2 + h.o.t \\ k_6 & = pn_1 + p^2 n_2 + h.o.t \end{array}\right\} \tag{5}$$

where h.o.t means "higher order terms in p" which is beyond the scope of the present study. By substituting Equations (4) and (5) into Equation (3) and collecting the same power of p by equating coefficients of p on LHS to RHS, then:

$$\begin{array}{ll} \dfrac{d}{dt}(v_o + pv_1 + p^2 v_2) & = pb_1 + p^2 b_2 - (pa_1 + p^2 a_2)(v_o + pv_1 + p^2 v_2) \\ & -\dfrac{(ph_1 + p^2 h_2)(z_o + pz_1 + p^2 z_2)(v_o + pv_1 + p^2 v_2)}{(z_o + pz_1 + p^2 z_2) + (pq_1 + p^2 q_2)} \\ & -(pi_1 + p^2 i_2)(x_o + px_1 + p^2 x_2)(v_o + pv_1 + p^2 v_2) \\ & +(pf_1 + p^2 f_2)(y_o + py_1 + p^2 y_2) \end{array} \tag{6}$$

such that for the coefficient of p^0 there exists

$$\frac{d}{dt}v_0 = 0 \tag{7}$$

and, $v_0(t) = v_0$ (constant)

For coefficient of p

$$\frac{d}{dt}v_1 = b_1 - a_1 v_0 - \frac{h_1 z_0 v_0}{z_1 + q_1} - i_1 v_0 x_0 + f_1 y_0 \tag{8}$$

For coefficient of p^2

$$\begin{aligned}\frac{d}{dt}v_2 \quad &= b_2 - (a_1 v_1 + a_2 v_0) - \frac{h_1 z_1 v_0 + h_1 z_0 v_1 + h_2 z_0 v_0}{z_2 + q_2} \\ &- (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) + (f_2 y_0 + f_1 y_1)\end{aligned} \tag{9}$$

Similarly

$$\begin{aligned}\frac{d}{dt}(x_o + p x_1 + p^2 x_2) \quad &= \frac{(ph_1 + p^2 h_2)(z_o + p z_1 + p^2 z_2)(v_o + p v_1 + p^2 v_2)}{(z_o + p z_1 + p^2 z_2) + (p q_1 + p^2 q_2)} \\ &+ (p i_1 + p^2 i_2)(x_o + p x_1 + p^2 x_2)(v_o + p v_1 + p^2 v_2) \\ &- (p l_1 + p^2 l_2)(x_o + p x_1 + p^2 x_2)\end{aligned} \tag{10}$$

For coefficient of p^0

$$\frac{d}{dt}x_0 = 0 \tag{11}$$

and, $x_0(t) = x_0$ (constant)

For coefficient of p

$$\frac{d}{dt}x_1 = \frac{h_1 z_0 v_0}{z_1 + q_1} - i_1 x_0 + i_1 x_0 v_0 \tag{12}$$

For coefficient of p^2

$$\begin{aligned}\frac{d}{dt}x_2 \quad &= \frac{h_1 z_0 v_1 + h_1 z_1 v_0 + h_2 z_0 v_0}{z_2 + q_2} \\ &+ (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) - (l_2 x_0 + l_1 x_1)\end{aligned} \tag{13}$$

Also,

$$\begin{aligned}\frac{d}{dt}(y_o + p y_1 + p^2 y_2) \quad &= (p e_1 + p^2 e_2)(x_o + p x_1 + p^2 x_2) \\ &- (p m_1 + p^2 m_2)(y_o + p y_1 + p^2 y_2)\end{aligned} \tag{14}$$

For coefficient of p^0

$$\frac{d}{dt}y_0 = 0 \tag{15}$$

and, $y_0(t) = y_0$ (constant)

For coefficient of p

$$\frac{d}{dt}y_1 = e_1 x_0 - m_1 y_0 \tag{16}$$

For coefficient of p^2

$$\frac{d}{dt}y_2 = e_1 x_1 + e_2 x_0 - (m_1 y_1 + m_2 y_0) \tag{17}$$

Lastly, using the fourth equation, then:

$$\begin{aligned} \frac{d}{dt}(z_o + pz_1 + p^2 z_2) &= (pj_1 + p^2 j_2)(x_o + px_1 + p^2 x_2) \\ &\quad - (pn_1 + p^2 n_2)(z_o + pz_1 + p^2 z_2) \end{aligned} \tag{18}$$

For coefficient of p^0

$$\frac{d}{dt}z_0 = 0 \tag{19}$$

and, $z_0(t) = z_0$ (constant)

For coefficient of p

$$\frac{d}{dt}z_1 = j_1 x_0 - n_1 z_0 \tag{20}$$

For coefficient of p^2

$$\frac{d}{dt}z_2 = (j_1 x_1 + j_2 x_0) - (n_1 z_1 + n_2 z_0) \tag{21}$$

By direct integration of equations (7), (11), (15) and (19) and using the initial conditions, then

$$\left.\begin{aligned} v_0(t) &= v_0 \\ x_0(t) &= x_0 \\ y_0(t) &= y_0 \\ z_0(t) &= z_0 \end{aligned}\right\} \tag{22}$$

where v_0, x_0, y_0, and z_0 are constants. By substituting Equation (22) into Equations (8), (9), (12), (13), (16), (17), (20), and (21) and simplifying using direct integration method, then:

From Equation (20)

$$\frac{d}{dt}z_1 = j_1 x_0 - n_1 z_0$$

integrating,

$$z_1(t) = k_7 t + c_1$$

where

$$k_7 = (j_1 x_0 - n_1 z_0)$$

and c_1 is the constant of integration.

$$z_1(0) = 0 \Rightarrow c_1 = 0$$

hence,

$$z_1(t) = k_7 t \quad \textbf{(23)}$$

From Equation (8) and by using Equation (23)

$$\frac{d}{dt}v_1 = b_1 - a_1 v_0 - \frac{h_1 z_0 v_0}{z_1 + q_2} - i_1 v_0 x_0 + f_1 y_0$$

$$\frac{d}{dt}v_1 = b_1 - a_1 v_0 - \frac{h_1 z_0 v_0}{k_7 t + q_2} - i_1 v_0 x_0 + f_1 y_0$$

Let $k_8 = b_1 - a_1 v_0 - i_1 v_0 x_0 + f_1 y_0$ and $k_9 = h_1 z_0 v_0$

then,

$$\frac{d}{dt}v_1 = k_8 - \frac{k_9}{k_7 t + q_2} \quad \textbf{(24)}$$

from which

$$dv_1 = \int \left\{ k_8 - \frac{k_9}{k_7 t + q_2} \right\} dt$$

On integration,

$$v_1(t) = k_8 t - \frac{k_9}{k_7} \ln(k_7 t + q_2) + K$$

With $v_1(0) = 0$

$$K = \frac{k_9}{k_7} \ln q_2$$

hence,

$$
\begin{aligned}
v_1(t) &= k_8 t - \frac{k_9}{k_7}\ln(k_7 t + q_2) + \frac{k_9}{k_7}\ln q_2 \\
v_1(t) &= k_8 t + \frac{k_9}{k_7}\ln\left(\frac{q_2}{k_7 t + q_2}\right)
\end{aligned}
$$

Therefore,

$$v_1(t) = k_{9b} t \qquad \textbf{(25)}$$

where

$$k_{9b} t = k_8 t + \frac{k_9}{k_7}\ln\left(\frac{q_2}{k_7 t + q_2}\right)$$

From Equation (12)

$$
\begin{aligned}
\frac{d}{dt}x_1 &= \frac{h_1 z_0 v_0}{z_1 + q_1} - i_1 x_0 + i_1 x_0 v_0 \\
\frac{d}{dt}x_1 &= \frac{k_9}{k_7 t + q_1} - k_{10} + k_{11},
\end{aligned}
$$

where

$$z_1 = k_7 t,\ k_{10} = i_1 x_0,\ k_{11} = i_1 x_0 v_0$$

then,

$$\frac{d}{dt}x_1 = \frac{k_9}{k_7 t + q_1} - k_{12}$$

where

$$k_{12} = k_{10} - k_{11}$$

then integrating,

$$x_1(t) = \frac{k_9}{k_7}\ln(k_7 t + q_1) - k_{12} t + c_2$$

and, with $x_1(0) = 0$

$$c_2 = -\frac{k_9}{k_7}\ln q_1$$

$$x_1(t) = \frac{k_9}{k_7}\ln\left(\frac{k_7 t + q_1}{q_1}\right) - k_{12} t$$

Therefore,

$$x_1(t) = k_{13} t \qquad \textbf{(26)}$$

where

$$k_{13}t = \frac{k_9}{k_7}\ln\left(\frac{k_7 t + q_1}{q_1}\right) - k_{12}t$$

From Equation (21)

$$\begin{aligned}
\frac{d}{dt}z_2 &= (j_1 x_1 + j_2 x_0) - (n_1 z_1 + n_2 z_0) \\
\frac{d}{dt}z_2 &= j_1 k_{13} t + j_2 x_0 - n_1 k_7 t - n_2 z_0 \\
\frac{d}{dt}z_2 &= (j_1 k_{13} - n_1 k_7)t + j_2 x_0 - n_2 z_0
\end{aligned}$$

Let

$$k_{14} = j_1 k_{13} - n_1 k_7,\ k_{15} = j_2 x_0 - n_2 z_0$$

then,

$$\frac{d}{dt}z_2 = k_{14}t + k_{15} \qquad \textbf{(27)}$$

On integration,

$$z_2(t) = \frac{k_{14}}{2}t^2 + k_{15}t + c_3$$

where c_3 is the constant of integration. Using $z_2(0) = 0 \Rightarrow c_3 = 0$ hence,

$$z_2(t) = \frac{k_{14}}{2}t^2 + k_{15}t \qquad \textbf{(28)}$$

From Equation (9)

$$\begin{aligned}
\frac{d}{dt}v_2 &= b_2 - (a_1 v_1 + a_2 v_0) - \frac{h_1 z_1 v_0 + h_1 z_0 v_1 + h_2 z_0 v_0}{z_2 + q_2} \\
&\quad - (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) + (f_2 y_0 + f_1 y_1)
\end{aligned}$$

Substituting Equations (23), (25), and (28) then,

$$\begin{aligned}
\frac{d}{dt}v_2 &= b_2 - (a_1 k_{9b} t + a_2 v_0) - \frac{h_1 k_7 t v_0 + h_1 z_0 k_{9b} t + h_2 z_0 v_0}{\frac{k_{14}}{2}t^2 + k_{15}t + q_2} \\
&\quad - (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) + (f_2 y_0 + f_1 y_1)
\end{aligned}$$

Let

$$\begin{aligned}
k_{16} &= b_2 - a_2 v_0 - (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) + (f_2 y_0 + f_1 y_1) \\
k_{17} &= h_1 k_7 t v_0 + h_1 z_0 k_{9b} t \\
k_{18} &= h_2 z_0 v_0 \\
k_{19} &= \frac{k_{14}}{2} \\
k_{20} &= a_1 k_{9b}
\end{aligned}$$

then $\frac{d}{dt}v_2$ becomes

$$\frac{d}{dt}v_2 = k_{16} - k_{20}t - \frac{k_{17}t + k_{18}}{k_{19}t^2 + k_{15}t + q_2}$$

If terms of $0(p^2)$ in the equation above is kept as in He (2006), then $k_{15} = k_{18}$ and $k_{17} = 2k_{19}$ such that,

$$dv_2 = \int \left\{ k_{16} - k_{20}t - \frac{k_{17}t + k_{18}}{k_{19}t^2 + k_{15}t + q_2} \right\} dt$$

On integration,

$$v_2(t) = k_{16}t - \frac{k_{20}t^2}{2} - \ln|k_{19}t^2 + k_{15}t + q_2| + c_4$$

where c_4 is the constant of integration. Using $v_2(0) = 0 \Rightarrow c_4 = \ln q_2$ hence,

$$v_2(t) = k_{16}t - \frac{k_{20}t^2}{2} + A \qquad \textbf{(29)}$$

where

$$A = \ln\left(\frac{q_2}{k_{19}t^2 + k_{15}t + q_2}\right)$$

From Equation (16)

$$\frac{d}{dt}y_1 = e_1x_0 - m_1y_0$$

Let

$$k_{21} = e_1x_0 - m_1y_0$$

and on integration,

$$y_1(t) = k_{21}t + c_5$$

where $c_5 = 0$ since $y_1(0) = 0$
hence,

$$y_1(t) = k_{21}t \qquad \textbf{(30)}$$

From Equation (17)

$$\begin{aligned}
\frac{d}{dt}y_2 &= (e_1x_1 + e_2x_0) - (m_1y_1 + m_2y_0) \\
\frac{d}{dt}y_2 &= e_1k_{13}t + e_2x_0 - m_1k_{21}t - m_2y_0 \\
\frac{d}{dt}y_2 &= (e_2x_0 - m_2y_0) + (e_1k_{13} - m_1k_{21})t
\end{aligned}$$

Let

$$k_{22} = (e_2 x_0 - m_2 y_0);\; k_{23} = (e_1 k_{13} - m_1 k_{21})$$

then,

$$\frac{d}{dt} y_2 = k_{23} t + k_{22}$$

On integration,

$$y_2(t) = \frac{k_{23}}{2} t^2 + k_{22} t + c_6$$

where $c_6 = 0$ since $y_2(0) = 0$ hence,

$$y_2(t) = \frac{k_{23}}{2} t^2 + k_{22} t \qquad \textbf{(31)}$$

From Equation (13)

$$\begin{aligned}\frac{d}{dt} x_2 \quad &= \frac{h_1 z_0 v_1 + h_1 z_1 v_0 + h_2 z_0 v_0}{z_2 + q_2} + (i_1 x_1 v_0 + i_2 x_0 v_0 + i_1 x_0 v_1) \\ &- (l_2 x_0 + l_1 x_1)\end{aligned}$$

By substituting for x_1, v_1, z_1, and z_2 then the above equation becomes

$$\begin{aligned}\frac{d}{dt} x_2 \quad &= \frac{h_1 z_0 k_{9b} t + h_1 k_7 t v_0 + h_2 z_0 v_0}{\frac{k_{14}}{2} t^2 + k_{15} t + q_2} + (i_1 k_{13} t v_0 + i_2 x_0 v_0 + i_1 x_0 k_{9b} t) \\ &- (l_2 x_0 + l_1 k_{13} t)\end{aligned}$$

Let

$$\begin{aligned} k_{24} \quad &= i_1 k_{13} t v_0 \\ k_{25} \quad &= i_2 x_0 v_0 \\ k_{26} \quad &= i_1 x_0 k_{9b} t \\ k_{27} \quad &= l_1 k_{13} t \\ k_{28} \quad &= l_2 x_0 \end{aligned}$$

then,

$$\frac{d}{dt} x_2 = (k_{24} + k_{26} - k_{27}) t + k_{25} - k_{28} + \frac{k_{17} t + k_{18}}{k_{19} t^2 + k_{15} t + q_2}$$

Also, let

$$\begin{aligned} k_{29} \quad &= (k_{24} + k_{26} - k_{27}) \\ k_{30} \quad &= (k_{25} - k_{28}) \end{aligned}$$

then,

$$\frac{d}{dt}x_2 = k_{29}t + k_{30} + \frac{k_{17}t + k_{18}}{k_{19}t^2 + k_{15}t + q_2}$$

$$dx_2 = \int\left\{k_{30} + k_{29}t + \frac{k_{17}t + k_{18}}{k_{19}t^2 + k_{15}t + q_2}\right\}dt$$

On integration,

$$x_2(t) = k_{30}t + \frac{k_{29}t^2}{2} + \ln|k_{19}t^2 + k_{15}t + q_2| + c_7$$

where c_7 is the constant of integration. Using $x_2(0) = 0 \Rightarrow c_7 = -\ln q_2$ hence,

$$x_2(t) = k_{30}t + \frac{k_{29}t^2}{2} + B \qquad \textbf{(32)}$$

where

$$B = \ln\left(\frac{k_{19}t^2 + k_{15}t + q_2}{q_2}\right)$$

By substituting Equations (22), (23), (25), (26), (28), (29), (30), (31) and (32) into Equation (4), the solutions of the system of Equation (3) are obtained by Parameter Expansion Method (PEM) as:

$$\begin{aligned}
v(t) &= v_0 + pk_{9b}t + p^2k_{16}t - p^2\frac{k_{20}}{2}t^2 + p^2\ln\left(\frac{q_2}{k_{19}t^2 + k_{15}t + q_2}\right)\\
x(t) &= x_0 + pk_{13}t + p^2k_{30}t + p^2\frac{k_{29}}{2}t^2 + p^2\ln\left(\frac{k_{19}t^2 + k_{15}t + q_2}{q_2}\right)\\
y(t) &= y_0 + pk_{21}t + p^2\frac{k_{23}}{2}t^2 + p^2k_{22}t\\
z(t) &= z_0 + pk_7t + p^2\frac{k_{14}}{2}t^2 + p^2k_{15}t
\end{aligned} \qquad \textbf{(33)}$$

In Equation (33), $p = 1$ (Sweilam, and Khader 2010).

RESULTS AND DISCUSSION

Validation studies have been omitted in this paper due to space constraint. Results of such computations can be found in some of the cited references (Nofal et al., 2013; Swilam and Al-Bar, 2009; Swilam and Khader, 2010), where parameter expansion method had been applied to model various types of physics and engineering phenomena. However, it is hoped that this discourse provides the basic knowledge necessary for applying parameter expansion method to model infectious diseases.

CONCLUSION

It is a known fact that systems of nonlinear ordinary differential equations are difficult to solve analytically but, in this study, we have formulated a nonlinear cholera model and a step-by-step approach has been adopted to obtain the analytical solutions of the model by using parameter expansion method. We therefore conclude that parameter expansion method is a suitable tool for the analysis of epidemic models.

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ABOUT THE AUTHORS

A.A. Ayoade, a registered teacher and a member of Mathematical Association of Nigeria (MAN), was born at Ile-Ife in 1977. He has been teaching mathematics for more than a decade and is one of those who are privileged to teach mathematics at all level of education-primary, secondary and tertiary. He earned his Ph.D. in Mathematics from University of Ilorin and is now a Lecturer in the Department of Mathematics, Kola Daisi University, Ibadan, Oyo State, Nigeria. His research interests are in the areas of biomathematics and numerical analysis. He has a number of published research articles in reputed national and international journals to his credit.

O.J. Peter, is a Ph.D. student in the Department of Mathematics, University of Ilorin, Nigeria. He holds a Master of Technology (M.Tech.) in Mathematics from Federal University of Technology Minna, Nigeria. His research interests are in mathematical modeling and numerical analysis.

S. Amadiegwu, is a Lecturer in the Department of Mathematics, School of General Studies, Maritime Academy of Nigeria, Oron, Akwa Ibom State. Nigeria. He holds a Master of Science (M.Sc.) in Mathematics from the University of Ibadan, Ibadan, Nigeria. His research interests are in biomathematics.

A.A. Victor, is a Ph.D. student in the Department of Mathematics, University of Ilorin, Nigeria. He holds a Master of Science (M.Sc.) in Mathematics from the same University. His research interests are in mathematical modeling and numerical analysis.

A.I. Abioye, is a Ph.D. student in the Department of Mathematics, University of Ilorin, Nigeria. He holds a Master of Science (M.Sc.) in Mathematics from the University of Lagos, Akoka, Nigeria. His research interests are in mathematical modeling.

A.F. Adebisi, is a Lecturer in the Department of Mathematical Sciences Osun State University Oshogbo, Nigeria. She hold Ph.D., M.Sc., and B.Sc. from University of Ilorin, Ilorin, Nigeria. Her research interests are in numerical analysis and mathematical modeling.

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