Estimating the Time of Death using the Lump System Analysis

Gbadebo I. Olatona and Abduljelili O. Popoola

Department of Physics, Osun State University, Osogbo, Nigeria.

E-mail: latonagbadebo@yahoo.com
popoolaabduljeleel2000@gmail.com

ABSTRACT

Imagine the number of innocent people that have been wrongly punished, sentenced, or even killed by courts of law or the police in murder cases due to inadequate knowledge about the time of death. Thus the determination of time of death is the subject of discussion in this paper. The model for estimating the time of death was obtained using the lump system analysis involving the energy balance between energy content of a body and energy loss by the body to the surroundings due to convection. An algorithm and computer program that computes the time of death written in C++ was done by the authors. The methodology developed in this paper could be used by coroners, the police and courts of law in investigating murder cases.

(Keywords: time of death, lump system analysis, forensic science, heat transfer, coroner, police)

INTRODUCTION

Estimating the time since death is an essential ingredient of every medicolegal postmortem examination (Dalbir et al., 2005) especially in Nigeria where security agents still rely on scared or bias witnesses to solve crimes. Just as Patricia Leinbach (Leinbach et al., 2015) wrote in her research work:

"The scene is a familiar one to any person who has watched modern mystery movies. The coroner is called to the witness stand and is questioned by the detectives investigating a murder case. At what time did the victim die? The coroner replies, 'between the hours of 10:45 and 11:00 p.m. on the night of…!'"

The time of death is therefore important piece of evidence in homicide investigation. In a less developed country like Nigeria, almost every passer-by around the area where a murder was committed are most always arrested with the aim of fishing out the killer out of hundreds of suspects. Meanwhile, the killer could have fled, leading to the punishment of innocent citizens. Estimating the time since death is therefore very important not only to rule suspects in or out but to also determine the causative agents or person and to tie in cause with time frame of illness or injury.

Mathematics is not the only solution as rightly put by Leinbach et al (2015); witnesses to the victim expiring, phone records, a stopped watch on the victim of an accidental death, death within medical facilities, etc. are also means of estimating the time since death but all these are obviously subjected to accidental error even though some of them can be very accurate. Meanwhile, a professional assassin would not kill in any of these circumstances and leave any trace in crime scene.

Many researchers have published regarding this topic such as the earliest temperature based model which emphasized that bodies will cool at 1^0C per hour (Leinbach et al., 2015). The model requires just the use of one temperature which is very easy to obtain on the crime scene and calculations involved are very simple but less accurate.

Many research works in this area such as Rainy's approach (Rainy, 1868 as quoted by Leinbach, 2015) and the research of Marshall and Hoare (Marshall and Hoare, 1962 as quoted by Leinbach, 2015) also make use of the Newton's law of cooling but almost none of them take into consideration the area of the human body probably because less work has been carried out in that field. This may render their works less accurate since the surface area of the body radiating the heat energy is not being put into consideration.

The method applied is the lump system analysis. For a system whose internal temperature changes with time only, the system is referred to as “lump”. In this system, temperature is not a function of position at all. So, it remains essentially constant with respect to position. The criterion for the application of this analysis is for the Biot number to be less or equal to 0.1. This however does not mean the analysis cannot be used for Biot number greater than 0.1. In fact, the analysis is approximate for Bi>0 and exact for Bi=0 (Peles, 2015).

Therefore, this paper is aimed at estimating the time of death using the lump system analysis considering the conditions of the human body including surface area, volume, thermal conductivity, density, etc. A model for estimating the time of death shall be generated and a computer program for estimating the time of death will be written so the time of death can be obtained using the parameters from the deceased and not only the crime scene.

METHODOLOGY

In heat transfer analysis, some bodies are observed to behave like a “lump” whose interior temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, T(t). Heat transfer analysis that utilizes this idealization is known as lumped system analysis, which provides great simplification in certain classes of heat transfer problem without much sacrifice from accuracy (Peles, 2015).

Consider a dead body of mass m, volume v, density ρ, specific heat capacity c, surface area A_s, initially at temperature T_i. At time t=0, the body is placed in a medium of temperature T_m and heat transfer takes place between the body and the medium with heat transfer coefficient h.
Applying the lump system analysis so that the body temperature remains constant with position at all times. Therefore, temperature at time t is T(t).

During a small change in time, dt, the temperature of the body decreases by a small amount, dT, and the medium rises in temperature by an equal amount, dT. An energy balance between the dead body and its medium therefore is given by:

Heat content of the body = Heat loss by convection by the body to its surrounding medium

Heat lost by conduction is neglected since air which is the surrounding fluid is a bad conductor of heat and heat lost by radiation is also neglected.

$$-mc_p \frac{dT}{dt} = hA_s\left(T - T_m\right) \qquad \mathbf{1}$$

Clearing the fraction in the last equation gives:

$$-mc_p dT = hA_s\left(T - T_m\right)dt \qquad \mathbf{2}$$

Note that: m = **ρ**v and dT = d(T_m-T) since T_m is a constant. Rearranging the last equation gives:

$$\frac{dT}{T_m - T} = \frac{hA_s}{mc_p} dt \qquad \mathbf{3}$$

Substituting dT = d(T_m-T) gives:

$$\frac{d\left(T_m - T\right)}{T_m - T} = \frac{hA_s}{mc_p} dt \qquad \mathbf{4}$$

Integrating the L.H.S for T_i at which t=0 to T(t) at time t:

$$In\left[T_m - T\right]_{Ti}^{T(t)} = \frac{hA_s}{mc_p}\left[t\right]_0^t \qquad \mathbf{5}$$

Evaluating the limits:

$$In\left[T_m - T(t)\right] - In\left[T_m - T_i\right] = \frac{hA_s}{mc_p} t \qquad \mathbf{6}$$

Rearranging the last equation gives:

$$In\frac{\left(T_m - T(t)\right)}{\left(T_m - T_i\right)} = \frac{hA_s}{mc_p} t \qquad \mathbf{7}$$

Therefore, making t the subject of equation in the last equation gives:

$$t = \frac{mc_p}{hA_s} In\left[\frac{T_m - T(t)}{T_m - T_i}\right] \qquad \mathbf{8}$$

t is the time interval between the time the person died and the time the body was found. The above can be written as shown below since T_m-T_i represents dT which can either be negative or positive. This is done to avoid logarithm of negative number:

$$t = \frac{mc_p}{hA_s} In\left[\frac{T_m - T(t)}{T_i - T_m}\right] \quad \textbf{9}$$

t is in seconds.

Let the time the person died be τ and the time the body was found b t_f. Therefore, the time of death is calculated as:

$$\tau = t - t_f \quad \textbf{10}$$

Applying 24-hour clock time: if τ is positive, then the time is p.m and a.m if τ is negative. Therefore the time of death on the clock is given as:

$$\tau = \frac{mc_p}{hA_s} In\left[\frac{T_m - T(t)}{T_i - T_m}\right] - t_f \quad \textbf{11}$$

The above is the model for estimating the time of death.

PARAMETERS COMPILATION

The values of m, T_m and T(t) can be measured from the crime scene. T_i can be taken as the average of room and body temperature. Considering the properties of the human body and the surrounding, others parameters like As, Cp and h can be obtained.

Calculation of the Heat Transfer Coefficient, h

The heat transfer coefficient, h can be calculated from the equation given below:

$$h = 1.42(\Delta T / L)^{1/4} \text{ (Dale et al, 2011)} \quad \textbf{12}$$

ΔT is the temperature difference between the body and the surrounding and L is the length of the specimen which in this case is the length of the dead body.

Since the average human body is 72 percent water by mass (Peles, 2015), we can therefore assume the body to take the properties of water. Therefore, the specific heat capacity is approximated as 4178J/Kg/K.

Calculation of the Body Surface Area, A_s

There are many formulas for estimating the body surface area ranging from Meeh-Boyd, Du Bois, Boyd, Du Bois-Boyd, etc., many of which can be very difficult to apply. However, in this work, we shall make use of formula proposed by Costef (1966), due to its flexibility and ease of use. Therefore, the body surface area:

$$A_s = \frac{4W + 7}{W + 90} \text{ (Costef, 1966)} \quad \textbf{13}$$

The model already had to do with the measurement of mass; it might therefore be time-consuming measuring the weight of the body again. Therefore, we shall use $W = 9.8m$ where m is the mass of the body and 9.8 is the acceleration due to gravity. This implies:

$$A_s = \frac{39.2m + 7}{9.8m + 90} \quad \textbf{14}$$

All quantities are in standard units.

Algorithm for Estimating the Time of Death

- Measure the mass of the dead body.
- Obtain the total surface area of the body from Equation 13.

$$A_s = \frac{39.2m + 7}{9.8m + 90} \quad \textbf{15}$$

- Measure the total length of the body.
- Obtain the difference in temperature between the body and the surrounding medium.
- Calculate the heat transfer coefficient h from Equation 12.

$$h = 1.42(\Delta T / L)^{1/4}.$$

- Measure the temperature of the surrounding medium T_m and the temperature at which the body was found T(t).

- Assuming that the person's normal temperature was not badly affected before death, then the normal body temperature can be applied such that T_i is constant at 37^0C.

- Taking the specific heat capacity of the body as 4187J/Kg/K.

- Measure and record the time the body was found, $t_{f.}$

- The time of death can therefore be obtained after substituting all the parameters in the model

$$\tau = \frac{mc_p}{hA_s} In\left[\frac{T_m - T(t)}{T_i - T_m}\right] - t_f$$

APPLICATION

Considering a body and an environment with the following parameters:

- Change in temperature between the body and the surrounding, ΔT =4.120C

- Total length of the body, L=1.61m

- Measure the mass of the dead body m=70.12kg

- The temperature of the surrounding medium T_m=27.34^{0}C

- The temperature at which the body was found T(t)=22.21^{0}C

- The time the body was found, t_f =5a.m which is just 5.00 applying 24-hour clock time

By substituting the above parameters in the model and computing using the computer program below, compiled and ran with devcpp.499 software, the following result was obtained:

A_s=3.54579m^2
Time of death= -0.676052

RESULT AND DISCUSSION

The result shows that the person did not die in the morning as the body was found around 5a.m in the morning. The negative indicates that the person died during p.m. hours. In this case, the person died around 12 – 0.676052 = 11.323948 p.m. which is approximately 11.00 p.m. Therefore, an investigator working on this type of case should widen the time range; say 11p.m to 1.00a.m, when investigating suspects. The computed body surface area is not a subject of discussion in this paper.

CONCLUSION

A model, an algorithm, and of course a program that estimates the time of death has been developed in this paper. It should however be understood that the product of this work is only an estimation and other evidence will have to be used to support the time of death. As it was said earlier, the lump system analysis is only approximate for Bi>0. So, the time of death is also an approximate value. It will only be exact for Bi=0 which is very impossible especially for a fairly conductive body like human body. Thus, an investigator using this model will have to use other evidence to support his findings not because this model is less accurate but because the investigator needs to be exact. The model gives the investigator an interval to begin questioning the whereabouts of the suspects and other material evidences.

RECOMMENDATION

The police, the coroner, the Federal Bureau of Investigation, courts of law and specialists are advised to employ this model in their investigations. The police especially are advised to employ this model in their investigation to reduce the rate at which innocents are being punished.

THE PROGRAM

```
#include<conio.h>
#include<iostream.h>
#include<stdlib.h>
#include<cmath>
#include<fstream>
#pragma hdrstop
int main () {
        float
r7,h7,l9,b9,h9,l6,b6,h6,l5,b5,h5,L,dT,h,Tm,
m,Tt,tf,A1,A2,A3,t_of_death;

        float As;
        const int Ti=31;
        const int cp=4187;
                // All values are in standard
units
                cout<<"Please enter some
variables before program execution.\n";
                  cout<<"\n\t Enter L (lenght
of the body):______\b\b\b\b";
                  cin>> L;
                  cout<<"\n\t Enter dT (temp
                  diff. b/w the body and the
                  surrouding):______\b\b\b\
                  b";
                  cin>> dT;
                cout<<"\n\t Enter Tm (temp of
the environment):______\b\b\b\b";
                  cin>> Tm;
                cout<<"\n\t Enter m (mass of
the body):______\b\b\b\b";
                  cin>> m;
                cout<<"\n\t Enter Tt (temp at
time found):______\b\b\b\b";
                  cin>> Tt;
                cout<<"\n\t Enter tf (time
found-24-hour time):______\b\b\b\b";
                  cin>> tf;

As=((39.2*m)+7)/((9.8*m)+90);
                h=1.42*pow((dT/L),0.25);

t_of_death=((((m*cp)/(h*As))*log((Tm-
Tt)/(Ti-Tm)))/3600)-tf;
                cout<<"As="<<As<<endl;

cout<<"t_of_death="<<t_of_death<<end
l;
                cout<<"Program finished!!!";
                cin.get();
                getch ();
    return 0; }
```

REFERENCES

1. Bentz D.P., A. Forster, K. Rice, M. Riley. 2011. “Thermal Properties and Thermal Modeling of Ballistic Clay Box”. National Institute of Standards and Technology (NIST): Gaithersburg, MD.

2. Costeff, H.A. 1966. “A Simple Empirical Formular for Calculating Approximate Surface Area in Children”. *Arch. Dis. Child*. 41:681.

3. Dalbir, S., S.K. Sharma, I.J. Dewan, Y.S. Bansal, D. Vishal, and A.N. Pandeya, 2005. “Polynomial Regression Model to Estimate Time Since Death in Adults from Rectal Temperature in Chandigarh zone of North West India”. *JIAFM*. 27(3). ISSN 0971 -0973.

4. Drillis, R.H., R. Contini, and M. Bluestein “Body Segment Parameters. A Survey of Measurement Technique”. *Artificial Limbs*. 8(1):44-66.

5. Leinbach, L.P. and L.C. Leinbach. 2000. “Estimating Time Since Death”. John Moores University: Liverpool, UK. Forth International Conference Derive TI 89/92, July 12-15, 2000.

6. Marshall, T.K. and F.E. Hoare. 1962. “Estimating the Time of Death: The Rectal Cooling after Death and its Mathematical Expression”. *Journal of Forensic Sciences.* 7(1):56–81.

7. Peles, Y. 2015. “Transient Heat Conduction”. In: Heat and Mass Transfer: Fundamentals and Applications”. Y. Cengel and A. Ghajar (eds). McGraw Hill: New York, NY.

8. Zatsiorsky, V.M., V.N. Seluyanov, and L.G. Chugunova. 1990. “Methods of Determining Mass Inertial Characteristics of Human Body Segments”. In: G.G. Chernyi and S.A. Regirer. *Contemporary Problems of Biomechanics.* CRC Press: Boca Raton, FL. Pp. 272-291.

SUGGESTED CITATION

Olatona, G.I. and A.O. Popoola. 2018. "Estimating the Time of Death using the Lump System Analysis". *Pacific Journal of Science and Technology*. 19(1):90-95.

Pacific Journal of Science and Technology