Some Robust and Classical Nonparametric Procedures of Estimations in Linear Regression Model

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ABSTRACT

We examined some robust and nonparametric procedures for a simple linear regression model when the error terms are drawn from unit normal, lognormal, Student t-10df, Cauchy, and exponential power via Monte Carlo simulation technique. The results showed that the nonparametric Theil's method demonstrates the strongest performance gains in many cases which imply they have negligible bias and the smallest Mean Square Error (MSE). The second best results are obtained from Least Trimmed Squares (LTS) Methods with relative reductions in MSE.

Though Least Absolute Deviation (LAD) and weighted Theil's regressions gave the poorest slope estimates with negative root mean square error (RMSE) values in most cases, it is noticed that LAD is always more efficient than LSE whenever the error component is Cauchy distributed. The LSE proved to be more efficient under normality assumption where as LTS and Theil's estimators performed better under non-normal data conditions.

(Keywords: least absolute deviation, least trimmed squares Monte Carlo simulation, ordinary least squares, regression analysis Theil's non-parametric, weighted Theil's non-parametric procedures)

INTRODUCTION

Regression analysis is a statistical tool for the investigation of relationships between variables. Usually, the investigator seeks to ascertain the causal effect of one variable upon another—the effect of a price increase upon demand, for example, or the effect of changes in the money supply upon the inflation rate. To explore such issues, the investigator assembles data on the underlying variables of interest and employs regression to estimate the *quantitative* effect of the causal variables upon the variable that they influence. The investigator also typically assesses the "statistical significance" of the estimated relationships, that is, the degree of confidence that the true relationship is close to the estimated relationship.

The simple linear regression model, which is the simplest form of a linear regression model, containing only one independent variable, is the focus of this research work. The independent variables, x_i's are assumed to be *non-stochastic* design values. Plus the standard theory assumption that the error terms come from normal distribution, we also deal with cases where the error terms have a lognormal distribution, Cauchy distribution, Exponential Error distribution and t10 (Student-t with 10df) distribution.

It is recognized that in the presence of normally distributed errors and homoscedasticity, OLS estimation is the method of choice. For situations in which the underlying assumptions of OLS estimation are not tenable, however, the choice of method for parameter estimation is not clearly defined. This research work reviews and briefly describes two classical nonparametric approaches and two robust regression approaches to Simple Linear Regression. The slope and *y*-intercept coefficients $\hat{\alpha}$, and $\hat{\beta}$ for the model (1)

$$I = \alpha + \beta E + \varepsilon \qquad \textbf{(1)}$$

are estimated by using the ordinary least squares (OLS), Least Absolute Deviation (LAD), Least Trimmed Squares and Theil's (and weighted

Theil) non parametric estimation methods. Variance, bias, mean square error, and relative mean square error of the estimates $\hat{\alpha}$ and $\hat{\beta}$ are used to evaluate estimators' performances with respect to each method under various situations.

MATERIALS AND METHODS

Ordinary Least Squares (OLS) Estimation Procedure

One of the most popular methods to model the functional relationship between variables is the OLS estimation procedure which is very simple and straightforward to apply. The basic idea of ordinary least squares is to optimize the fit by minimizing the total sum of the squares of the errors (deviations) between the observed values y_i and the estimated values $\hat{\alpha} + \hat{\beta}x$. In other words, the OLS method estimates the parameters $\hat{\alpha}$ and $\hat{\beta}$ that minimize the sum of squares of the residuals $S(\hat{\alpha},\hat{\beta})$, which is given in Equation (1).

Taking the partial derivative of (1) with respect to $\hat{\alpha}$ and $\hat{\beta}$, and equating to zero we get the OLS normal equations, the values of $\hat{\alpha}$ and $\hat{\beta}$ that satisfy the equations are given by:

$$\hat{\beta} = \frac{\sum_i^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_i^n (x_i - \bar{x})^2},$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

For testing hypothesis and constructing confidence intervals, the random errors, ε_i's are assumed to be normally and independently distributed which leads to normally distributed response variables, y_i' s (Rawlings et. al., 1998). The properties of OLS estimators are stronger under the normality assumption. If the error terms are normal and identically and independently distributed (i.i.d) with zero mean and constant variance S^2, then the OLS estimators $\hat{\alpha}$ and $\hat{\beta}$ attain uniformly minimum variance in the range of all unbiased estimators. Under these assumptions the likelihood function of ε_i's is:

$$L \propto \left(\frac{1}{\sigma}\right)^n exp\left\{-\frac{1}{2\sigma^2}\sum_i^n (y_i - \hat{\alpha} - \hat{\beta}x_i)^2\right\}$$

Under normality, the solutions of the equations given below are the maximum likelihood (ML) estimators which are also equal to the OLS estimators:

$$\frac{\partial lnL}{\partial \hat{\alpha}} = \frac{1}{\sigma^2}\sum_i^n (y_i - \hat{\alpha} - \hat{\beta}x_i)^2 = 0$$

$$\frac{\partial lnL}{\partial \hat{\beta}} = \frac{1}{\sigma^2}(y_i - \hat{\alpha} - \hat{\beta}x_i)x_i = 0$$

$$\frac{\partial lnL}{\partial \sigma} = -\frac{n}{\sigma^2} + \frac{1}{\sigma^3}\sum_i^n (y_i - \hat{\alpha} - \hat{\beta}x_i)^2 = 0$$

These assumptions and several types of model deficiencies can be detected with the help of the residual analysis. Suppose that the distribution of the errors is not normal. If the errors are coming from a population that has a mean of 0, then the OLS estimates may not be optimal, but they at least have the property of being unbiased. If we further assume that the variance of the error population is finite, then the OLS estimates have the property of being consistent and asymptotically normal. However, under these conditions, the OLS estimates and tests may lose much of their efficiency and they can result in poor performance. To deal with these situations, two approaches can be applied. One is to try to correct non-normality, if non-normality is determined and the other is to use alternative regression methods, which do not depend on the assumption of the normality (Birkes and Dodge, 1993).

Alternative Methods to OLS

In real life, it is difficult to find a data set that satisfies all the assumptions necessary to apply OLS method. It has a 0% breakdown value, which means that a small percentage of contamination can cause the estimators to take values from -∞ to +∞ (Rousseeuw and Leroy, 1987). Hence, if the observations are not normally distributed or they contain outliers, the OLS method is no longer convenient. That is why robust regression procedures are needed to remove the adverse effect of these situations. An estimator is said to be robust if it is fully efficient (nearly so) under an assumed model but maintains high efficiency for possible alternatives. Any robust method must be reasonably efficient when compared to the least squares estimators;

if the underlying distribution of errors are independent normal, and substantially more efficient than least squares estimators, when there are outlying observations.

In this study, we investigated four other alternatives to OLS; Least Absolute Deviation, Least Trimmed Squares, Theil's Pairwise-Median and weighted Theil's non parametric procedure.

Least Absolute Deviation

The Least Absolute Deviations regression (LAD regression) is one of the principal alternatives to the Ordinary Least Squares method when one seeks to estimate regression parameters. The goal of the LAD regression is to provide a robust estimator which minimized the sum of the absolute residuals min$\sum_{i=1}^{n}|r_i|$. For a fixed β, the α, which minimizes the *f* function below is the sample median of $\{y_i - \beta x_i\}$.

$$f(\alpha,\beta) = \sum_{i=1}^{n}|\hat{y}_i - (\alpha + \beta x_i)|,$$

The *Y*-outliers have less impact on the LAD results, because it does not square the residuals, and then the outliers are not given as much weight as in OLS procedure. However, LAD regression estimator is just as vulnerable as least squares estimates to high leverage outliers (X-outliers).

Least Trimmed Squares (LTS) Estimator

LTS is an estimation procedure which achieves the purpose of being insensitive to changes in small percentage of data points. It aims at minimizing $\sum_{i=1}^{h}(\hat{y}_i - \hat{\alpha} - \hat{\beta}x_i)^2$. To minimize the Function 1, we should choose a subsample of *h* observations and compute some α and β that minimize the sum of squared residuals for the selected subsample. By applying this procedure to all subsamples, we have $\binom{n}{h}$ estimates for both α and β and the estimate which makes the objective function smallest is the final estimate (Cizek and Visek, 2000). Unfortunately, it is very difficult to obtain all subsamples unless a very small sample is analyzed.

In LTS procedure, data points corresponding to a specified percentage of the largest residuals under an initial OLS estimation are deleted. The outlying cases are deleted to reduce their adverse effect on the estimators (Nevitt and Tam, 1998). Thus, the only difference between OLS and LTS estimation is that in LTS, the largest squared residuals are not used (n-h observations will not affect the estimator). It has been demonstrated that the best robustness properties are achieved when h is approximately n/2, in which case the breakdown point attains 50% (Rousseeuw and Leroy, 1987).

Theil's and Weighted Theil's Non-parametric Procedures

The robust estimate of slope for nonparametric fitted line was first described by Theil (1950). Theil's regression is a nonparametric method which is used as an alternative to robust methods for data sets with outliers. Although the non-parametric procedures perform reasonably well for almost any possible distribution of errors and they lead to robust regression lines, they require a lot of computation. It is proved to be useful when outliers are suspected, but when there are more than few variables, the application becomes difficult (Mutan, 2004).

Theil (1950) proposed two methods, namely, the complete (Theil) and the weighted Theil method. The complete Theil slope estimate is computed by comparing each data pair to all others in a pairwise fashion. A data set of n(X,Y) pairs will result in N = $\binom{n}{2} = \frac{n(n-1)}{2}$ pairwise comparisons. For each of these comparisons a slope $\frac{\Delta Y}{\Delta X}$ is computed. The median of all possible pairwise slopes is taken as the nonparametric Thiel's slope estimate, β^{THEIL} Where:

$$b_{ij} = \frac{\Delta Y}{\Delta X} = \frac{y_j - y_i}{x_j - x_i} \quad x_j \neq x_i;\ 1 \leq i < j \leq n$$

All the x_i are assumed to be distinct, and we will lose no generality that they are arranged in ascending order (Hussain and Sprent, 1983).

Conover's estimator assures that the fitted line goes through the point (X_{median} ,Y_{median}). This is analogous to OLS, where the fitted line always goes through the point (x_i,y_i) (Nadia and Amaa, 2013).

To reduce the effect of outlying observations, some modifications are applied to Theil's method and each of the pairwise slopes, b_{ij}' s, are weighted by some weighting procedures. The weighted Theil slope estimator for the n observations in the sample data is the weighted median of these b_{ij}'s. Recall that $\hat{\beta}_{OLS} = \sum w_{ij}\, b_{ij}$ where $w_{ij} = \frac{(x_i - x_j)^2}{\sum (x_i - x_j)^2}$ and Σ represents $\frac{n(n-1)}{2}$ pairs of integers *i* and *j* with $1 \leq i < j \leq n\ n$.

According to Birkes and Dodge (1993) a weighted median can be calculated as follows: x_i's are ordered in an increasing sequence, so that $x_1 < x_2 < \ldots < x_n$. The index k is obtained such that:

$w_1 + w_2 + \ldots + w_{k-1} < 0.5$ or $W_1 + w_2 + \ldots + w_{k-1} + w_k > 0.5$

where the weights, w_i's, are nonnegative and add up to 1. x_k is the *weighted median*.

Thus the weighted Theil slope estimator of β is the pairwise slopes $b_{ij} = (y_i - y_j) / (x_i - x_j)$, with weights $w_{ij} = |x_i - x_j| / \Sigma|x_i - x_j|^2$, and $\hat{\alpha}_{(wtd.THL)}$ is the ordinary median of $(y_i - \hat{\beta}_{(wtd.THL)} * x_i)$

Monte Carlo Simulation

Suppose that a statistic T based on a sample $x_1, x_2, \ldots, x_n$ has been formulated for testing a certain hypothesis; the test procedure is to reject H_0 if $T(x_1, x_2, \ldots, x_n) \geq c$. Since the exact distribution of T is unknown, the value of c has been determined such that the asymptotic type I error rate is α = 0.05, (say), i.e., $\Pr(T \geq c|H_0 \text{ is true}) = .05$ as $n \to \infty$. We can study the actual small sample behavior of T by the following procedure:

a. Generate $x_1, x_2, \ldots, x_n$ from the appropriate distribution, say, the Normal distribution and compute $T_j = T(x_1, x_2, \ldots, x_n)$.

b. Repeat N times, yielding a sample $T_1, T_2, \ldots, T_N$, and

c. compute proportion of times $T_j \geq c$ as an estimate of the error rate
i.e., $\hat{\alpha}_N = \frac{1}{N}\sum_{j=1}^{N} I(T_j \geq c)$

d. Note that $\hat{\alpha}_N \xrightarrow{a.s.}$ α, the type I error rate.

Results of such a study may provide evidence to support the asymptotic theory even for small samples. Similarly, we may study the power of the test under different alternatives using Monte Carlo sampling. The study of Monte Carlo methods involves learning about:

(i) Methods available to perform step (a) above correctly and efficiently, and

(ii) How to perform step (c) above efficiently, by incorporating variance reduction" techniques to obtain more accurate estimates.

RESULTS AND DISCUSSIONS

The study addresses the problem via computer (Monte-Carlo) simulation methods. All programming for the simulation study is developed using FORTRAN. The design variable X is generated using a sequential model of the form $X_t = t$; $t = 1, 2, 3, \ldots, n$, while the response variable Y is generated using the relationship $Y_t = \alpha + \beta X_t + u_t$ (for α = 2 and β = 1).

The study design includes using N= (150000/n) replications of randomly generated samples each of three sample sizes (*n* = 10, 30, 100) crossed with five types of error distributions (unit normal, lognormal, *t*-10*df, Cauchy and exponential power*) from which the random component u_t is drawn. Alternative forms of the error distribution (mixture, contaminated and outliers) model are also considered to test the sensitivity of each of the simple linear regression estimation methods under study to various degrees and forms of outliers in the response variable Y direction. Algorithm for drawing random deviates from each of the error distributions are described in Evans, Hastings, and Peacock (1993).

For the alternative forms of the error distribution, taking the normal distribution as example: Standard model is a standard variate N(0,1). The Outliers Model is a mixture of N(0,1) and N(0,9) with probability p. The Mixture Model is a mixture of N(0,1) and N(5,9) with probability p while the Contamination Model is a mixture of N(0,1) and G(p) with probability p (where G(p) is a geometric random variate with pmf pq^n and probability 0.5). In each of the cases defined, p= 0.80. For detailed information on this approach see Evans, Hastings, and Peacock (1993).

For each simulated data set, the estimators of α and β are calculated. The estimation techniques that were considered are OLS, LAD, 20% LTS, Theil's and weighted Theil's regression. Using these procedures, the y-intercept α and slope β estimators are computed and for each estimator mean, variance, bias, mean square error (MSE) and relative mean square error (RMSE) are calculated.

Effects of Sample Size

Across sample sizes, estimator variances (and, to some lesser degree estimator bias) decreased with increasing sample size. For example, the variances for the LSE slope estimator under the standard unit normal distribution are 0.0122, 0.0004, 0.0001 and 0.000011 for sample sizes n = 10, 30, 50 and 100 respectively. Also, the variances for the Theil's slope estimator under the mixture model for the Cauchy distribution are 0.4113, 0.0021, 0.00038 and 0.000011 for sample sizes n = 10, 30, 50, and 100, respectively. This pattern of decreasing variance and bias holds for all estimators under all error distributions.

The patterns seen in the variances are also exhibited in the estimator MSE values. Because the results for the n = 30 sample size are intermediate to those for the n = 10 and n = 50 sample sizes, they are not reported here. It should also be noted that the exponential power distribution is very close to the unit normal distribution when its location parameter a=0 and scale and shape parameter equals one (b=c=1). Thus, the distribution was used only as a control for the normality assumption in LSE.

Slope Estimators Performances

Based On Bias Criterion: Table 1 gives summary results for β across all cells of the simulation study. A close look at this table reveals that LSE, LTS and Theil's slopes estimators are approximately unbiased i.e. they have negligible bias. This pattern of performance is observed to improve consistently as sample size increases and across all error distributions and respective alternative models. However, a deeper look at Table 1 reveals that LSE is always biased whenever the error term is Cauchy distributed regardless of sample size.

LAD and Weighted Theil's estimators are biased estimators of the population slope parameter β as they consistently over estimated β across all cells of the simulation study. However, LAD is observed to converge only when the sample size is relatively very large (n=100) or very small (n=10) and the error term is non-normal.

With the introduction of contamination into the dataset, it was observed that LSE, LTS and Theil's estimator still remained unbiased while LAD and Weighted Theil's estimators remained biased except under the Mixture Model where all the estimators are relatively biased across all error distributions for very small sample size (n=10). Under this condition, Theil's estimator stayed on top most of the time with the least bias value, followed closely by LTS and then LSE with LAD and Weighted Theil's estimators competing rigorously with each other at the tail end.

Based On Variance and RMSE Criteria: Table 2 and 3 give summary results for β based on the variance and Relative Mean Square Error criteria across all cells of the simulation study. A close look at this table reveals that LSE had the least variance and hence the least MSE under the normal distribution. As sample size increases, LSE gained precision i.e. the variance and MSE decreases with increasing sample size. For instance, at n=50, there was 99.18% decrease in both variance and MSE value of LSE and at n=100 there was 99.91% decrease in both variance and MSE value of LSE. Under the Student-t distribution, LSE gained precision with a 52.93% reduction in both variance and MSE when n=10 and 99.99% decrease in both variance and MSE when n=100 relative to normal distribution. But as the error term begins to deviate from normality, LSE was observed to consistently loose precision with almost 400% increase in MSE across all sample sizes under the lognormal distribution and for the Cauchy distribution, LSE gave an outrageously large value for MSE compare to other estimators. This pattern of behavior confirms the fact that deviations from normality cause OLS estimators to be poor estimators.

The Theil's nonparametric slope estimator followed LSE closely under the normal distribution with respect to MSE values (0.01380) but as sample size increases it gains precision and its MSE values (0.0001054 for n=50 and 0.00001240 for n=100) were approximately equal with that of LSE when the sample is very large (n≥50). Under the student-t distribution, it gained more precision with a 17.72% and 57.36% decrease in MSE relative to LSE when n=10 and n=100 respectively, thereby outperforming LSE. The same pattern of behavior was observed under the lognormal and Cauchy distribution. Hence, Theil's gave a positive RMSE values whenever the error term come from non-normal distributions (Table 3). This confirms that Theil estimator has high small-sample efficiency compared to the OLS estimator when the error term is heteroscedastic (Wilcox, 1998).

The Least Trimmed Squares estimation method came next in view to Theil's with a MSE value 0.01739 under the normal distribution for sample size n=10 and as has been usually observed, it experiences a consistent decrease in its MSE as sample size gets larger (0.0001331, n=50; 0.00001666, n=100). For the non-normal distribution cases, LTS compete so rigorously with Theil's with a 14.87% (n=10) and 41.82% (n=100) decrease in MSE relative to LSE under the student-t distribution, thus, outperforming LSE and hence giving a positive RMSE value. It was observed that the farther away the error distribution deviated from normality, the better LTS becomes as it gains precision (55.35% when n=10, 42.97% when n=100 under Lognormal distribution and 100% when n=10, 99.96% when n==100 under Cauchy distribution) following closely after Theil's while displacing LSE out-rightly.

A close look at the tables above showed that LAD is especially consistent with the other estimators (in terms of its variance) only when the sample size is either very small (n=10) or very large (n=100). However, its MSE does not really show a consistent pattern with regards to sample size due to the fact that LAD consistently underestimated the slope parameter β, thus giving a negative bias across all sample sizes and distribution. It is also worthy of note that LAD gave a bigger variance and MSE value than any other estimator (except weighted Theil) across all sample sizes and error distribution with the exception of Cauchy distribution under which LAD gave a MSE value that is significantly smaller than that of LSE (1.1380 vs 7978, 0.9546 vs 88.87, 1.045 vs 1.190 for n=10, 50, and 100, respectively) and therefore having a favorable RMSE across all sample sizes only when the error term is Cauchy distributed.

The weighted Theil's estimator, like LAD, is not a good competitor compared to other slope estimator since it gave an inconsistent estimate of variance and MSE across all sample sizes and error distributions. Unlike LAD though, it consistently loses precision as the error term deviates from normality and its MSE was always highest whenever the error term is Cauchy distributed.

Effects of Contamination

For the first case of contamination considered in this project (outliers case), It was observed that Theil's nonparametric slope estimator outperformed every other slope estimators regardless of sample size or distribution. LSE slope estimator was better than LTS when the sample size is small (n<50) and the error distribution is normal. LAD maintained its status-quo beating LSE only when the error distribution is Cauchy. Like LAD, weighted Theil's slope estimator remained inconsistent and also maintained its response pattern to sample size and error distribution. Unlike LAD and weighted Theil, LSE, LTS and Theil's slope estimators generally improved in their precision compared to the standard model case. More so, the intercept estimators' performance pattern remains the same for all estimators except for LTS, which like Theil's, proved to be more efficient than LSE, LAD and weighted Theil regardless of sample size or distribution (Table 3).

It is clear from Table 3 (which present results for RMSE) that as long as the error distribution is normal, LSE slope estimator win the game. But as the sample size gets larger (n≥100), Theil's nonparametric slope estimator defeats LSE by a slim chance of 4.37% reduction in its MSE relative to LSE. However, when the error is non-normal, Theil's always takes the crown followed closely by LTS and then LAD (as usual, whenever the error is Cauchy distributed).

The general pattern of estimators' behavior, for the contamination case is similar to that of outliers' error model case, only that for small samples (n≤30) and whenever the error term is normally distributed, LSE slope estimator is the most efficient of all. But whenever n≥50, Theil's nonparametric slope estimator takes the stage irrespective of distribution type and under non-normal error distribution situation, Theil's, LTS and LAD kept to their status-quo.

Table 1: Summary of Population Intercept (α) Estimators' Performance (Based on Bias).

ESTIMATION METHODS	**ERROR MODEL TYPE**			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	**OUTLIER MODEL**	**MIXTURE MODEL**	**CONTAMINATION MODEL**
NORMAL	[1]LTS (-0.0019), [2]LSE (-0.0026) [3]THEIL (-0.0029), [4]LAD (7.2850) [5]wTHEIL (-11.9800)	[1]THEIL (-0.0005), [2]LSE (-0.0005) [3]LTS (0.0021), [4]LAD (7.1900) [5]wTHEIL (11.2400)	[1]THEIL (0.1724), [2]TLE (0.1805) [3]LSE (0.1897), [4]LAD (7.3570) [5]wTHEIL (14.0500)	[1]LTS (-0.0013), [2]LSE (-0.0020) [3]THEIL (-0.0024), [4]LAD (7.3030) [5]wTHEIL (11.9600)
STUDENT-T (10df)	[1]THEIL (-0.0041), [2]LTS (-0.0053) [3]LSE (-0.0061), [4]wTHEIL (6.5470) [5]LAD (6.5470)	[1]THEIL (-0.0022), [2]LTS (-0.0029) [3]LSE (-0.0030), [4]wTHEIL (4.9450) [5]LAD (7.0520)	[1]LTS(0.0111), [2]LSE (0.0126) [3]THEIL (0.0129), [4]LAD (6.5690) [5]wTHEIL (6.5710)	[1]THEIL (-0.0033), [2]LSE (-0.0049) [3]LTS (-0.0049), [4]wTHEIL(6.2200) [5]LAD (6.5620)
LOGNORMAL	[1]THEIL (1.1350), [2]LTS (1.2040) [3]LSE (1.6360), [4]LAD (8.8090) [5]wTHEIL (25.5000)	[1]THEIL (0.9788), [2]LTS (0.9488) [3]LSE (1.1620), [4]LAD (8.9670) [5]wTHEIL (22.4700)	[1]THEIL (1.4610), [2]LTS (1.6040) [3]LSE (2.2460), [4]LAD (9.2440) [5]wTHEIL (34.9100)	[1]THEIL (1.0260), [2]LTS (1.0990) [3]LSE (1.5380), [4]LAD (8.6910) [5]wTHEIL (26.1500)
CAUCHY	[1]THEIL (-0.0267), [2]LTS (-0.0285) [3]LSE (6.6230), [4]LAD (7.2150) [5]wTHEIL (413.4000)	[1]THEIL (-0.0542), [2]LTS (0.1224) [3]LSE (-0.2572), [4]LAD (8.1040) [5]wTHEIL (748.6000)	[1]THEIL (0.1264), [2]LTS (1.2500) [3]LAD (9.3360), [4]LSE (78.4000) [5]wTHEIL (2603.0000)	[1]THEIL (-0.2095), [2]LTS (-0.3084) [3]LSE (-2.3260), [4]LAD (6.3360) [5]wTHEIL (153.9000)
SAMPLE SIZE (N=50)				
NORMAL	[1]THEIL (0.0000), [2]LTS (-0.0002) [3]LSE (-0.0015), [4]LAD (35.1700) [5]wTHEIL (81.0300)	[1]THEIL (-0.0014), [2]LTS (-0.0049) [3]LSE (-0.0078), [4]LAD (34.9800) [5]wTHEIL (77.9000)	[1]LTS (0.1465), [2]THEIL (0.1806) [3]LSE (0.1966), [4]LAD (-24.5400) [5]wTHEIL (90.13000)	[1]LTS (-0.0003), [2]THEIL (0.0009) [3]LSE (-0.0014), [4]LAD (20.2900) [5]wTHEIL (80.9800)
STUDENT-T (10df)	[1]LSE (-0.0031), [2]THEIL (-0.0045) [3]LTS (-0.0052), [4]LAD (28.7300) [5]wTHEIL (31.05)	[1]THEIL (-0.0002), [2]LSE (-0.0007) [3]LTS (-0.0011), [4]wTHEIL(16.4000) [5]LAD (25.4500)	[1]LTS (0.0043), [2]THEIL (0.0044) [3]LSE (0.0074), [4]LAD (26.0600) [5]wTHEIL (31.1200)	[1]LSE (-0.0024), [2]LTS (-0.0033) [3]THEIL (-0.0035), [4]LAD (20.0700), [5]wTHEIL (29.6300)
LOGNORMAL	[1]THEIL (1.0220), [2]LTS (1.1400) [3]LSE (1.6330), [4]LAD (29.8600) [5]wTHEIL (237.0000)	[1]LTS (0.9293), [2]THEIL (0.9965) [3]LSE (1.1840), [4]LAD (25.9000) [5]wTHEIL (216.3000)	[1]THEIL (1.2470), [2]LTS (1.3440) [3]LSE (2.2730), [4]LAD (23.9200) [5]wTHEIL (251.8000)	[1]THEIL (0.8713), [2]LTS (1.0230) [3]LSE (1.5300), [4]LAD (22.5500) [5]wTHEIL (90.13000)
CAUCHY	[1]THEIL (0.0015), [2]LTS (0.0318) [3]LSE (-3.0690), [4]LAD (30.4600) [5]wTHEIL (8750.4000)	[1]THEIL (0.0039), [2]LTS (0.1165) [3]LSE (-15.9700), [4]LAD (41.5300) [5]wTHEIL (23250.0000)	[1]THEIL (0.0016), [2]LTS (1.4500) [3]LSE (8.7060), [4]LAD (27.2000) [5]wTHEIL (58820.0000)	[1]THEIL (-0.1048), [2]LTS (-0.3940) [3]LSE (-4.5150), [4]LAD (29.8200) [5]wTHEIL (2928.0000)
SAMPLE SIZE (N=100)				
NORMAL	[1]THEIL (-0.0002), [2]LTS (-0.0009) [3]LSE (-0.0046), [4]LAD (538.6000) [5]wTHEIL (994.2000)	[1]THEIL (-0.0008), [2]LSE (-0.0028) [3]LTS (0.0031), [4]LAD (Nill) [5]wTHEIL (-173.3000)	[1]LTS (0.1148), [2]THEIL (0.1788) [3]LSE (0.1891), [4]LAD (18.7000) [5]wTHEIL (195.5000)	[1]LTS (0.0000), [2]THEIL (-0.0004) [3]LSE (-0.0048), [4]LAD (32.4900) [5]wTHEIL (178.5000)
STUDENT-T (10df)	[1]THEIL (0.0008), [2]LTS (0.0017) [3]LSE (0.0026), [4]LAD (60.2000) [5]wTHEIL (61.6200)	[1]LTS (0.0000), [2]THEIL (0.0000) [3]LSE (0.0002), [4]wTHEIL (26.3800) [5]LAD (43.0800)	[1]THEIL (0.0066), [2]LTS (0.0079) [3]LSE (0.0010), [4]LAD (48.4800) [5]wTHEIL (61.7100)	[1]THEIL (0.0006), [2]LTS (0.0018) [3]LSE (0.0027), [4]LAD (4.8280) [5]wTHEIL (58.9800)
LOGNORMAL	[1]THEIL (1.0120), [2]LTS (1.1310) [3]LSE (1.6370), [4]LAD (60.4200) [5]wTHEIL (621.6000)	[1]LTS (0.9368), [2]THEIL (0.9986) [3]LSE (1.2030), [4]LAD (46.6400) [5]wTHEIL (577.8000)	[1]THEIL (1.2220), [2]LTS (1.3060) [3]LSE (2.2530), [4]LAD (52.5000) [5]wTHEIL (620.0000)	[1]THEIL (0.8548), [2]LTS (1.0180) [3]LSE (1.5370), [4]LAD (20.6400) [5]wTHEIL (628.3000)
CAUCHY	[1]THEIL (-0.0112), [2]LTS (-0.0415) [3]LSE (-2.6980), [4]LAD (90.5600) [5]wTHEIL (25130.0000)	[1]LTS (0.0083), [2]THEIL (-0.0189) [3]LSE (-3.2330), [4]LAD (50.0300) [5]wTHEIL (51790.0000)	[1]THEIL (-0.0112), [2]LTS (1.3980) [3]LSE (15.7100), [4]LAD (48.5900) [5]wTHEIL (126100.0000)	[1]THEIL (-0.0865), [2]LTS (-0.4483) [3]LSE (-4.9740), [4]LAD (-131.9000) [5]wTHEIL (14350.0000)

Table 2: Summary Table for Population Intercept (α) Estimators' Performance (Based on Variance).

ESTIMATION METHODS	ERROR MODEL TYPE			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	OUTLIER MODEL	MIXTURE MODEL	CONTAMINATION MODEL
NORMAL	[1]THEIL (0.5583), [2]LSE (0.4651) [3]LTS (0.6621), [4]LAD (5.1880) [5]wTHEIL (18.0700)	[1]THEIL (0.0981), [2]LTS (0.1188) [3]LSE (0.1978), [4]LAD (5.5950) [5]wTHEIL (17.9700)	[1]THEIL (0.8084), [2]LSE (0.6289) [3]LTS (1.0640), [4]LAD (5.1910) [5]wTHEIL (19.4200)	[1]LSE (0.4621), [2]THEIL (0.5458) [3]LTS (0.6509), [4]LAD (5.4290) [5]wTHEIL (18.1500)
STUDENT-T (10df)	[1]THEIL (0.2722), [2]LTS (0.2698) [3]LSE (0.3043), [4]LAD (9.7540) [5]wTHEIL (11.2600)	[1]LTS (0.0205), [2]THEIL (0.0208) [3]LSE (0.0466), [4]wTHEIL (4.1640) [5]LAD (8.7040)	[1]LTS (0.2699), [2]THEIL (0.2719), [3]LSE (0.3035), [4]LAD (10.6700) [5]wTHEIL (11.3000)	[1]THEIL (0.2176), [2]LTS (0.2185) [3]LSE (0.2726), [4]LAD (10.4500) [5]wTHEIL (10.8800)
LOGNORMAL	[1]THEIL (0.6811), [2]LTS (1.0380) [3]LSE (2.1710), [4]LAD (6.5850) [5]wTHEIL (489.7000)	[1]THEIL (0.0932), [2]LTS (0.1960) [3]LSE (0.8810), [4]LAD (5.9160) [5]wTHEIL (426.6000)	[1]THEIL (1.6960), [2]LTS (3.1630) [3]LSE (3.4680), [4]LAD (6.430) [5]wTHEIL (395.3000)	[1]THEIL (1.7980), [2]LTS (1.1730) [3]LSE (2.2660), [4]LAD (6.907) [5]wTHEIL (490.1000)
CAUCHY	[1]THEIL (3.7770), [2]LTS (9.8550) [3]LAD (15.5100), [4]LSE (603500.00) [5]wTHEIL (262700000.0000)	[1]LAD (16.9700), [2]THEIL (24.2600) [3]LTS (267.3000), [4]LSE (826000.0000) [5]wTHEIL (289600000.0000)	[1]THEIL (17.8200), [2]LTS (354.8000) [3]LAD (1831.0000), [4]LSE (4854000.), [5]wTHEIL (20980000000.00)	[1]THEIL (1.3190), [2]LTS (3.5430) [3]LAD (45.1100), [4]LSE (4294.00.), [5]wTHEIL (3772000.00)
	SAMPLE SIZE (N=50)			
NORMAL	[1]LSE (0.0838), [2]THEIL (0.0978) [3]LTS (0.1118), [4]wTHEIL (275.6000) [5]LAD (460.7000)	[1]THEIL (0.0018), [2]LTS (0.0193) [3]LSE (0.0346), [4]wTHEIL (292.3000) [5]LAD (345.6000)	[1]LSE (0.1129), [2]THEIL (0.1448) [3]LTS (0.2245), [4]wTHEIL (233.7000) [5]LAD (1557.0000)	[1]THEIL (0.0830), [2]LSE (0.0836) [3]LTS (0.1106), [4]LAD (276.5000) [5]wTHEIL (368.6000)
STUDENT-T (10df)	[1]THEIL (0.0113), [2]LTS (0.0139) [3]LSE (0.0197), [4]LAD (110.1000) [5]wTHEIL (201.8000)	[1]THEIL (0.0001), [2]LTS (0.0007) [3]LSE (0.0019), [4]LAD (12.0100) [5]wTHEIL (19.3200)	[1]THEIL (0.0123), [2]LTS (0.0139) [3]LSE (0.0196), [4]LAD (5.3910) [5]wTHEIL (202.7000)	[1]THEIL (0.0070), [2]LTS (0.0099) [3]LSE (0.0176), [4]wTHEIL (191.700) [5]LAD (679.9000)
LOGNORMAL	[1]THEIL (0.0798), [2]LTS (0.1772) [3]LSE (0.3873), [4]LAD (110.3000) [5]wTHEIL (22750.0000)	[1]THEIL (0.0014), [2]LTS (0.0291) [3]LSE (0.1612), [4]LAD (40.41000) [5]wTHEIL (2054.0000)	[1]THEIL (0.1642), [2]LTS (0.3354) [3]LSE (0.6156), [4]LAD (75.9200) [5]wTHEIL (17930.0000)	[1]THEIL (0.0991), [2]LTS (0.2120) [3]LSE (0.4070), [4]LAD (75.9600) [5]wTHEIL (22760.0000)
CAUCHY	[1]THEIL (0.2968), [2]LTS (2.7800) [3]LAD (53.7200), [4]LSE (18640.0000) [5]wTHEIL (29060000000.0000)	[1]THEIL (1.6430), [2]LTS (129.7000) [3]LSE (666400.00), [4]LAD (1335.0000) [5]wTHEIL (114300000000)	[1]THEIL (0.1642), [2]LTS (0.132.3) [3]LAD (141.5000), [4]LSE (867500.00) [5]wTHEIL (2268000000000.000)	[1]THEIL (0.1271), [2]LTS (0.9392) [3]LAD (165.8000), [4]LSE (8026.000) [5]wTHEIL (1102000000.0000)
	SAMPLE SIZE (N=100)			
NORMAL	[1]LSE (0.0396), [2]THEIL (0.0475) [3]LTS (0.0572), [4]LAD (538.6000) [5]wTHEIL (994.2000)	[1]THEIL (0.0004), [2]LTS (0.0093) [3]LSE (0.0179), [4]LAD (Nill) [5]wTHEIL (811.2000)	[1]LSE (0.0532), [2]THEIL (0.0692) [3]LTS (0.1151), [4]LAD (517.8000) [5]wTHEIL (817.9000)	[1]THEIL (0.0355), [2]LSE (0.0395) [3]LTS (0.0564), [4]LAD (501.6000) [5]wTHEIL (997.3000)
STUDENT-T (10df)	[1]THEIL (0.0030), [2]LTS (0.0041) [3]LSE (0.0063), [4]LAD (67.2200) [5]wTHEIL (811.2000)	[1]THEIL (0.0000), [2]LTS (0.0002) [3]LSE (0.0005), [4]LAD (156.9000) [5]wTHEIL (37.1900)	[1]THEIL (0.0030), [2]LTS (0.0041) [3]LSE (0.0062), [4]LAD (8.6240) [5]wTHEIL (810.6000)	[1]THEIL (0.0016), [2]LTS (0.0029) [3]LSE (0.0057), [4]wTHEIL (746.700) [5]LAD (969.8000)
LOGNORMAL	[1]THEIL (0.0380), [2]LTS (0.1018) [3]LSE (0.1945), [4]LAD (2.2170) [5]wTHEIL (151400.0000)	[1]LAD (0.0000), [2]THEIL (0.0003) [3]LTS (0.0155), [4]LSE (0.0902) [5]wTHEIL (145600.0000)	[1]THEIL (0.0741), [2]LTS (0.1349) [3]LSE (0.3090), [4]LAD (10.7400) [5]wTHEIL (140500.0000)	[1]THEIL (0.0453), [2]LTS (0.1208) [3]LSE (0.2012), [4]LAD (255.8000) [5]wTHEIL (150900.0000)
CAUCHY	[1]THEIL (0.1261), [2]LTS (1.4510) [3]LAD (140.8000), [4]LSE (6693.0000) [5]wTHEIL (40110000000.0000)	[1]THEIL (0.7097), [2]LAD (11.0900) [3]LTS (27.4200), [4]LSE (25150.0000) [5]wTHEIL (71470000000.0000)	[1]LAD (0.0000), [2]THEIL (0.1277) [3]LTS (35.2900), [4]LSE (54860.0000) [5]wTHEIL (670600000000.0000)	[1]THEIL (0.0527), [2]LTS (0.7406) [3]LSE (6146.00), [4]LAD (195400.00) [5]wTHEIL (32490000000.0000)

Table 3: Summary Table for Population Intercept (α) Estimators' Performance (Based on RMSE)

ESTIMATION METHODS	ERROR MODEL TYPE			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	OUTLIER MODEL	MIXTURE MODEL	CONTAMINATION MODEL
NORMAL	[1]LSE (-), [2]THEIL (-0.2005) [3]LTS (-0.4236), [4]LAD (-124.3000.), [5]wTHEIL (-346.5000)	[1]THEIL (0.5040), [2]LTS (0.3995) [3]LSE (-), [4]LAD (-288.7000) [5]wTHEIL (-729.2000)	[1]LSE (-), [2]THEIL (-0.2550) [3]LTS (-0.6494), [4]LAD (-88.2000.), [5]wTHEIL (-324.9000)	[1]LSE (-), [2]THEIL (-0.1811) [3]LTS (-0.4085), [4]LAD (-126.1000.), [5]wTHEIL (-347.5000)
STUDENT-T (10df)	[1]LTS (0.1134), [2]THEIL (0.1054) [3]LSE (-), [4]LAD (-16.37000) [5]wTHEIL (-176.9000)	[1]LTS (0.5607), [2]THEIL (0.5549) [3]LSE (-), [4]wTHEIL (-612.6000) [5]LAD (-1253.0000)	[1]LTS (0.1109), [2]THEIL (0.1039) [3]LSE (-), [4]LAD (-176.2000) [5]wTHEIL (-178.4000)	[1]THEIL (0.2020), [2]LTS (0.1984) [3]LSE (-), [4]wTHEIL (-180.8000) [5]LAD (-195.3000)
LOGNORMAL	[1]THEIL (0.5936), [2]LTS (0.4865) [3]LSE (-), [4]LAD (-16.3700) [5]wTHEIL (-234.2000)	[1]THEIL (0.5288), [2]LTS (0.5093) [3]LSE (-), [4]LAD (-37.6900) [5]wTHEIL (-416.5000)	[1]THEIL (0.5500), [2]LTS (0.3263) [3]LSE (-), [4]LAD (-9.7910) [5]wTHEIL (-188.6000)	[1]THEIL (0.6007), [2]LTS (0.4858) [3]LSE (-), [4]LAD (-16.8000) [5]wTHEIL (-252.4000)
CAUCHY	[1]THEIL (1.0000), [2]LTS (1.0000) [3]LAD (0.9999), [4]LSE (-) [5]wTHEIL (-434.5000)	[1]THEIL (1.0000), [2]LAD (0.9999) [3]LTS (0.9997), [4]LSE (-) [5]wTHEIL (-350.2000)	[1]THEIL (1.0000), [2]LAD (1.0000) [3]LTS (1.0000), [4]LSE (-) [5]wTHEIL (-431.3000)	[1]THEIL (0.9997), [2]LTS (0.9992) [3]LAD (0.9802), [4]LSE (-) [5]wTHEIL (-881.7000)
	SAMPLE SIZE (N=50)			
NORMAL	[1]LSE (-), [2]THEIL (-0.1675) [3]LTS (-0.3343), [4]LAD (-20260.000), [5]wTHEIL (-81640.0000)	[1]THEIL (0.9468), [2]LTS (0.4433) [3]LSE (-), [4]LAD (-45230.0000) [5]wTHEIL (-183300.0000)	[1]LSE (-), [2]THEIL (-0.1616) [3]LTS (-0.6107), [4]wTHEIL (-54710), [5]LAD (-105900.0000)	[1]THEIL (0.0070), [2]LSE (-) [3]LTS (-0.3227), [4]LAD (-9332.000), [5]wTHEIL (-81720.0000)
STUDENT-T (10df)	[1]THEIL (0.4266), [2]LTS (0.2943) [3]LSE (-), [4]LAD (-47400.0000) [5]wTHEIL (-59070.0000)	[1]THEIL (0.9579), [2]LTS (0.6191) [3]LSE (-), [4]LAD (-351900.7000) [5]wTHEIL (-153800.0000)	[1]THEIL (0.4279), [2]LTS (0.2919) [3]LSE (-), [4]LAD (-34740.0000) [5]wTHEIL (-59460.0000)	[1]THEIL (0.6004), [2]LTS (0.4379) [3]LSE (-), [4]LAD (-61600.0000) [5]wTHEIL (-60860.0000)
LOGNORMAL	[1]THEIL (0.6316), [2]LTS (0.5166) [3]LSE (-), [4]LAD (-327.1000) [5]wTHEIL (-25805.0000)	[1]LTS (0.4293), [2]THEIL (0.3643) [3]LSE (-), [4]LAD (-453.7000) [5]wTHEIL (-43050.0000)	[1]THEIL (0.7028), [2]LTS (0.6296) [3]LSE (-), [4]LAD (-111.1000) [5]wTHEIL (-14060.0000)	[1]THEIL (0.6876), [2]LTS (0.5418) [3]LSE (-), [4]LAD (-211.8000) [5]wTHEIL (-29280.0000)
CAUCHY	[1]THEIL (1.0000), [2]LTS (0.9999) [3]LAD (0.9474), [4]LSE (-) [5]wTHEIL (-1562000.000)	[1]THEIL (1.0000), [2]LTS (0.9998) [3]LAD (0.9954), [4]LSE (-) [5]wTHEIL (-172200.000)	[1]THEIL (1.0000), [2]LTS (0.9998) [3]LAD (0.9990), [4]LSE (-) [5]wTHEIL (-2618000.0000)	[1]THEIL (1.0000), [2]LTS (0.9999) [3]LAD (0.8689), [4]LSE (-) [5]wTHEIL (-138000.0000)
	SAMPLE SIZE (N=100)			
NORMAL	[1]LSE (-), [2]THEIL (-0.1976) [3]LTS (-0.4434), [4]LAD (-20930.000), [5]wTHEIL (-829600.0000)	[1]THEIL (0.9793), [2]LTS (0.4792) [3]LSE (-), [4]LAD (-54.9800) [5]wTHEIL (-1739000.0000)	[1]LSE (-), [2]THEIL (-0.1369) [3]LTS (-0.4418), [4]LAD (-10350), [5]wTHEIL (-438800.0000)	[1]THEIL (0.1008), [2]LSE (-) [3]LTS (-0.4260), [4]LAD (-39400.00), [5]wTHEIL (-831300.0000)
STUDENT-T (10df)	[1]THEIL (0.5220), [2]LTS (0.3479) [3]LSE (-), [4]LAD (-590000.0000) [5]wTHEIL (-737200.0000)	[1]THEIL (0.9884), [2]LTS (0.6580) [3]LSE (-), [4]wTHEIL (-1548000.0000) [5]LAD (-4247.0000)	[1]THEIL (0.5214), [2]LTS (0.3396) [3]LSE (-), [4]LAD (-375200.0000) [5]wTHEIL (-734800.0000)	[1]THEIL (0.7144), [2]LTS (0.4977) [3]LSE (-), [4]LAD (-174700.0000) [5]wTHEIL (-743300.0000)
LOGNORMAL	[1]THEIL (0.6303), [2]LTS (0.5191) [3]LSE (-), [4]LAD (-1270.0000) [5]wTHEIL (-187200.0000)	[1]LTS (0.4187), [2]THEIL (0.3507) [3]LSE (-), [4]LAD (-1415.0000) [5]wTHEIL (-312100.0000)	[1]THEIL (0.7091), [2]LTS (0.6582) [3]LSE (-), [4]LAD (-512.6000) [5]wTHEIL (-97420.0000)	[1]THEIL (0.6973), [2]LTS (0.5483) [3]LSE (-), [4]LAD (-265.0000) [5]wTHEIL (-212900.0000)
CAUCHY	[1]THEIL (1.0000), [2]LTS (0.9998) [3]LSE (-), [4]LAD (-0.2450) [5]wTHEIL (-6080000.0000)	[1]THEIL (1.0000), [2]LTS (0.9989) [3]LAD (0.9001), [4]LSE (-) [5]wTHEIL (-2947000.0000)	[1]THEIL (1.0000), [2]LTS (0.9993) [3]LAD (0.9572), [4]LSE (-) [5]wTHEIL (-12460000.0000)	[1]THEIL (1.0000), [2]LTS (0.9998) [3]LSE (-), [4]LAD (-33.4800) [5]wTHEIL (-5299000.0000)

Table 4: Summary Table for Population Slope (β) Estimators' Performance (Based on Bias).

ESTIMATION METHODS	ERROR MODEL TYPE			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	**OUTLIER MODEL**	**MIXTURE MODEL**	**CONTAMINATION MODEL**
NORMAL	^{1}LTS (0.0000), ^{2}LSE (0.0001) ^{3}THEIL (0.0003), ^{4}LAD (-0.9967) ^{5}wTHEIL (-2.1780)	^{1}LSE (-0.0002), ^{2}THEILS (-0.0002) ^{3}LTS (-0.0005), ^{4}LAD (-0.9912) ^{5}wTHEIL (-2.0440)	^{1}THEIL (0.1724), ^{2}LTS (0.1805) ^{3}LSE (0.1897), ^{4}LAD (7.3570) ^{5}wTHEIL (14.0500)	^{1}THEIL (-0.0000), ^{2}LSE (0.0000), ^{3}LTS (-0.0002), ^{4}LAD (-0.9899), ^{5}wTHEIL (2.1740)
STUDENT-T (10df)	^{1}THEIL (0.0006), ^{2}LSE (0.0009) ^{3}LTS (0.0009), ^{4}LAD (-0.9170) ^{5}wTHEIL (-1.1900)	^{1}LTS (0.0005), ^{2}THEIL (0.0005), ^{3}LSE (-0.0005), , ^{4}wTHEIL (-0.8987) ^{5}LAD (-0.9285)	^{1}LTS (0.0111), ^{2}LSE (0.0126), ^{3}THEIL (0.0129), ^{4}LAD (6.5690) ^{5}wTHEIL (6.5710)	^{1}THEIL (0.0005), ^{2}LSE (0.0007) ^{3}LTS (0.0008), ^{4}LAD (-0.8836) ^{5}wTHEIL (-1.1310)
LOGNORMAL	^{1}THEIL (0.0002), ^{2}LTS (-0.0004) ^{3}LSE (0.0020), ^{4}LAD (-0.9805) ^{5}wTHEIL (-4.3370)	^{1}THEIL (0.0170), ^{2}LTS (0.0318) ^{3}LSE (-0.0647), ^{4}LAD (-1.0010) ^{5}wTHEIL (-3.7850)	^{1}THEIL (1.4610), ^{2}LTS (1.6040) ^{3}LSE (2.2460), ^{4}LAD (9.2440) ^{5}wTHEIL (34.9100)	^{1}THEIL (-0.0010), ^{2}LSE (0.0013) ^{3}LTS (-0.0014), ^{4}LAD (-0.9837) ^{5}wTHEIL (-4.4740)
CAUCHY	^{1}THEIL (0.0042), ^{2}LTS (0.0066) ^{3}LSE (-0.8266), ^{4}LAD (-1.0040) ^{5}wTHEIL (-74.8900)	^{1}THEIL (0.0076), ^{2}LTS (-0.0143) ^{3}LSE (0.0908), ^{4}LAD (-1.1280) ^{5}wTHEIL (-135.8000)	^{1}THEIL (0.1264), ^{2}LTS (1.2500) ^{3}LAD (9.3360), ^{4}LSE (78.4000) ^{5}wTHEIL (2603.0000)	^{1}THEIL (0.0010), ^{2}LTS (0.0018) ^{3}LSE (-0.1825), ^{4}LAD (-0.9939) ^{5}wTHEIL (-28.4500)
	SAMPLE SIZE (N=50)			
NORMAL	^{1}LTS (0.0000), ^{2}THEIL (0.0000) ^{3}LSE (0.0001), ^{4}wTHEIL (-3.1770) ^{5}LAD (NILL)	^{1}THEIL (0.0001), ^{2}LTS (0.0001) ^{3}LSE (0.0003), ^{4} wTHEIL (-3.0540) ^{5} LAD (Nill)	^{1}THEIL (-0.0000), ^{2} LSE (0.0000) ^{3} LTS (-0.0001), ^{4}LAD (0.2472) ^{5}wTHEIL (-3.5260)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0001), ^{4}LAD (-0.4250) ^{5}wTHEIL (-3.1750)
STUDENT-T (10df)	^{1}LSE (0.0001), ^{2}THEIL(0.0001) ^{3}LTS (0.0002), ^{4}wTHEIL (-1.2180) ^{5}LAD (NILL)	^{1}THEIL (0.0000), ^{2}LSE (0.0000) ^{3}LTS (0.0000), ^{4}LAD (-0.9788) ^{5}wTHEIL (-0.6431)	^{1}LTS (0.0000), ^{2}THEIL (-0.0000) ^{3}LSE (-0.0001), ^{4}LAD (-0.9997) ^{5}wTHEIL (-1.2200)	^{1}LSE (0.0001), ^{2}THEIL (0.0001) ^{3}LTS (0.0001), ^{4}LAD (-0.1857) ^{5}wTHEIL (-1.1620)
LOGNORMAL	^{1}THEIL (0.0001), ^{2}LTS (0.0002) ^{3}LSE (0.0005), ^{4}LAD (-0.9923) ^{5}wTHEIL (-9.2300)	^{1}THEIL (0.0005), ^{2}LTS (0.0069) ^{3}LSE (0.0130), ^{4}LAD (-0.9857) ^{5}wTHEIL (-8.4190)	^{1}THEIL (0.0000), ^{2}LSE (0.0003) ^{3}LTS (0.0003), ^{4}LAD (-0.7432) ^{5}wTHEIL (-9.7820)	^{1}THEIL (0.0001), ^{2}LTS (0.0002) ^{3}LSE (0.0005), ^{4}LAD (-0.7084) ^{5}wTHEIL (-9.358)
CAUCHY	^{1}THEIL (-0.0000), ^{2}LTS (-0.0006) ^{3}LSE (0.2259), ^{4}LAD (-0.9648) ^{5}wTHEIL (-343.2000)	^{1}THEIL (-0.0001), ^{2}LTS (-0.0029) ^{3}LSE (0.5904), ^{4}LAD (-0.8946) ^{5}wTHEIL (-911.8000)	^{1}THEIL (-0.0001), ^{2}LTS (0.0042) ^{3}LAD (-0.7533), ^{4}LSE (1.5700) ^{5}wTHEIL (-2306.0000)	^{1}THEIL (-0.0000), ^{2}LTS (-0.0005) ^{3}LSE (0.0578), ^{4}LAD (-0.9581) ^{5}wTHEIL (-114.9000)
	SAMPLE SIZE (N=100)			
NORMAL	^{1}LTS (-0.0000), ^{2}THEIL (0.0001) ^{3}LSE (0.0000), ^{4} LAD (-0.9516) ^{5}wTHEIL (-3.5360)	^{1}LTS (-0.0000), ^{2}THEIL (0.0000) ^{3}LSE (0.0001), ^{4}wTHEIL (-3.4320) ^{5} LAD (Nill)	^{1}THEIL (0.0001), ^{2}LSE (0.0002) ^{3}LTS (0.0002), ^{4} LAD (-0.0920) ^{5}wTHWIL (-3.8690)	^{1}LTS (-0.0000), ^{2}THEIL (0.0000) ^{3}LSE (0.0001), ^{4}LAD (-0.8394) ^{5}wTHEIL (-1.1680)
STUDENT-T (10df)	^{1}THEIL (-0.0000), ^{2}LTS (-0.0000) ^{3}LSE (-0.0000), ^{4} LAD (-1.1690) ^{5}wTHEIL (-1.2210)	^{1}LSE (-0.0000), ^{2}LTS (-0.0000) ^{3}THEIL (-0.0000), ^{4}wTHEIL (-0.5226) ^{5}LAD (-0.9044)	^{1}THEIL (-0.0001), ^{2}LTS (-0.0001) ^{3}LSE (-0.0001), ^{4}LAD (-0.4160) ^{5}wTHEIL (-1.2220)	^{1}THEIL (-0.0000), ^{2}LTS (-0.0000) ^{3}LSE (-0.0000), ^{4}LAD (0.9302) ^{5}wTHEIL (-1.1680)
LOGNORMAL	^{1}THEIL (0.0001), ^{2}LTS (0.0001) ^{3}LSE (0.0002), ^{4}LAD (-1.1690) ^{5}wTHEIL (-12.2800)	^{1}THEIL (0.0001), ^{2}LTS (0.0033) ^{3}LSE (0.0063), ^{4}LAD (-0.8967) ^{5}wTHEIL (-11.4100)	^{1}THEIL (0.0001), ^{2}LTS (0.0003) ^{3}LSE (-0.0006), ^{4}LAD (-0.7739) ^{5}wTHEIL (-12.2300)	^{1}LTS (0.0000), ^{2}THEIL (-0.0000) ^{3}LSE (0.0000), ^{4}LAD (1.5600) ^{5}wTHEIL (-12.4100)
CAUCHY	^{1}THEIL (0.0002), ^{2}LTS (0.0004) ^{3}LSE (0.0517), ^{4}LAD (-1.0210) ^{5}wTHEIL (-497.5000)	^{1}LTS (-0.0001), ^{2}THEIL (0.0003) ^{3}LSE (0.0592), ^{4}LAD (-0.9493) ^{5}wTHEIL (-1025.0000)	^{1}THEIL (0.0002), ^{2}LTS (0.0003) ^{3}LSE (0.2744), ^{4}LAD (-0.9032) ^{5}wTHEIL (-2498.0000)	^{1}LTS (0.0000), ^{2}THEIL (0.0001) ^{3}LSE (0.0239), ^{4}LAD (-0.3281) ^{5}wTHEIL (-284.3000)

Table 5: Summary Table for Population Slope (β) Estimators' Performance (Based on Variance).

ESTIMATION METHODS	ERROR MODEL TYPE			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	**OUTLIER MODEL**	**MIXTURE MODEL**	**CONTAMINATION MODEL**
NORMAL	^{1}LSE (0.0122), ^{2}THEIL (0.0138) ^{3}LTS (0.0174), ^{4}LAD (0.0733) ^{5}wTHEIL (0.5808)	^{1}THEIL (0.0065), ^{2}LSE (0.0070) ^{3}LTS (0.0075), ^{4}LAD (0.0836) ^{5}wTHEIL (0.5764)	^{1}LSE (0.0164), ^{2}THEIL (0.0190 ^{3}LTS (0.0276), ^{4}LAD (0.0820) ^{5}wTHEIL (0.6198)	^{1}LSE (0.0121), ^{2}THEIL (0.0136) ^{3}LTS (0.0171), ^{4}LAD (0.0914) ^{5}wTHEIL (0.5832)
STUDENT-T (10df)	^{1}THEIL (0.0047), ^{2}LTS (0.0049) ^{3}LSE (0.0057), ^{4}LAD (0.1514) ^{5}wTHEIL (0.3701)	^{1}THEIL (0.0009), ^{2}LTS (0.0009) ^{3}LSE (0.0011), ^{4}LAD (0.1256) ^{5}wTHEIL (0.1363)	^{1}THEIL (0.0047), ^{2}LTS 0.0049) ^{3}LSE (0.0057), ^{4}LAD (0.1620) ^{5}wTHEIL (0.3714)	^{1}THEIL (0.0039), ^{2}LTS (0.0040) ^{3}LSE (0.0051), ^{4}LAD (0.1628) ^{5}wTHEIL (0.3574)
LOGNORMAL	^{1}THEIL (0.0157), ^{2}LTS (0.0265) ^{3}LSE (0.0593), ^{4}LAD (0.1005) ^{5}wTHEIL (15.3000)	^{1}THEIL (0.0077), ^{2}LTS (0.0136) ^{3}LSE (0.0259), ^{4}LAD (0.1033) ^{5}wTHEIL (13.1400)	^{1}THEIL (0.0354), ^{2}LTS (0.0820) ^{3}LSE (0.0930), ^{4}LAD (0.1028) ^{5}wTHEIL (12.1800)	^{1}THEIL (0.0180), ^{2}LTS (0.0298) ^{3}LSE (0.0619), ^{4}LAD (0.1007) ^{5}wTHEIL (15.3100)
CAUCHY	^{1}THEIL (0.1037), ^{2}LTS (0.2325) ^{3}LAD (0.1300), ^{4}LSE (7978.0) ^{5}wTHEIL (8671000.0)	^{1}LAD (0.2757), ^{2}THEIL (0.4928) ^{3}LTS (4.7250), ^{4}LSE (11900) ^{5}wTHEIL (9560000.00)	^{1}THEIL (0.4113), ^{2}LAD (0.8005) ^{3}LTS (7.9060), ^{4}LSE (630500) ^{5}wTHEIL (692400000.00)	^{1}THEIL (0.0340), ^{2}LTS (0.0832) ^{3}LAD (0.1417), ^{4}LSE (196.4000) ^{5}wTHEIL (126100.00)
	SAMPLE SIZE (N=50)			
NORMAL	^{1}LSE (0.00010), ^{2}THEIL (0.00011) ^{3}LTS (0.00013), ^{4}wTHEIL (0.4232) ^{5}LAD (NILL)	^{1}THEIL (0.0000), ^{2}LTS (0.0001) ^{3}LSE (0.0001), ^{4}wTHEIL (0.4490) ^{5}LAD (NILL)	^{1}LSE (0.0001), ^{2}THEIL (0.0001) ^{3}LTS (0.0003), ^{4}wTHEIL (0.3588) ^{5}LAD (7.2230)	^{1}THEIL (0.0001), ^{2}LSE (0.0001) ^{3}LTS (0.0001), ^{4}LAD (0.3771) ^{5}wTHEIL (0.4246)
STUDENT-T (10df)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}wTHEIL (0.3103) ^{5}LAD (NILL)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (0.0038) ^{5}wTHEIL (0.0298)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (0.0063) ^{5}wTHEIL (0.3116)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (2.1490) ^{5}wTHEIL (0.2948)
LOGNORMAL	^{1}THEIL (0.0008), ^{2}LTS (0.0002) ^{3}LSE (0.0005), ^{4}LAD (0.0008) ^{5}wTHEIL (34.9200)	^{1}THEIL (0.0000), ^{2}LTS (0.0001) ^{3}LSE (0.0003), ^{4}LAD (0.0018) ^{5}wTHEIL (31.5000)	^{1}THEIL (0.0001), ^{2}LTS (0.0004) ^{3}LSE (0.0007), ^{4}LAD (0.3076) ^{5}wTHEIL (27.5200)	^{1}THEIL (0.0001), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (2.1490) ^{5}wTHEIL (0.2948)
CAUCHY	^{1}THEIL (0.0004), ^{2}LTS (0.0038) ^{3}LAD (0.0240), ^{4}LSE (88.8200) ^{5}wTHEIL (44690000)	^{1}THEIL (0.0015), ^{2}LAD (0.0853) ^{3}LTS (0.1005), ^{4}LSE (597.4000) ^{5}wTHEIL (175700000)	^{1}THEIL (0.0004), ^{2}LAD (0.1106) ^{3}LTS (0.1936), ^{4}LSE (6681.4000) ^{5}wTHEIL (3488000000)	^{1}THEIL (0.0002), ^{2}LTS (0.0011) ^{3}LAD (0.1409), ^{4}LSE (6.4210) ^{5}wTHEIL (1694000)
	SAMPLE SIZE (N=100)			
NORMAL	^{1}LSE (0.0000), ^{2}THEIL (0.0000) ^{3}LTS (0.0000), ^{4} LAD (0.0039) ^{5}wTHEIL (0.3886)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}wTHEIL (0.4046) ^{5}LAD (Nill)	^{1}THEIL (0.0000), ^{2}LSE (0.0000) ^{3}LTS (0.0000), ^{4}wTHEIL (0.3199) ^{5}LAD (1.0360)	^{1}THEIL (0.0000), ^{2}LSE (0.0000) ^{3}LTS (0.0000), ^{4}LAD (0.0636) ^{5}wTHEIL (0.3899)
STUDENT-T (10df)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4} LAD (0.0333) ^{5}wTHWIL (0.3181)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (0.0007) ^{5}wTHEIL (0.01459)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}LAD (0.2431) ^{5}wTHEIL (0.3179)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0000), ^{4}wTHEIL (0.2928) ^{5}LAD (2.5850)
LOGNORMAL	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0001), ^{4}LAD (0.0012) ^{5}wTHEIL (59.3500)	^{1}LAD (0.0000), ^{2}THEIL (0.0000) ^{3}LTS (0.0000), ^{4}LSE (0.0000) ^{5}wTHEIL (57.0800)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0001), ^{4}LAD (0.1551) ^{5}wTHEIL (55.0500)	^{1}THEIL (0.0000), ^{2}LTS (0.0000) ^{3}LSE (0.0001), ^{4}LAD (2.2480) ^{5}wTHEIL (59.1500)
CAUCHY	^{1}THEIL (0.0000), ^{2}LTS (0.0004) ^{3}LAD (0.0028), ^{4}LSE (1.1870) ^{5}wTHEIL (15730000)	^{1}THEIL (0.0002), ^{2}LAD (0.0027) ^{3}LTS (0.0055), ^{4}LSE (5.1200) ^{5}wTHEIL (28030000.0000)	^{1}LAD (0.0000), ^{2}THEIL (0.0000), ^{3}LTS (0.0096), ^{4}LSE (36.1300), ^{5}wTHEIL (263000000.0000)	^{1}THEIL (0.0000), ^{2}LTS (0.0002) ^{3}LAD (0.2617), ^{4}LSE (0.7575) ^{5}wTHEIL (12740000.0000)

Table 6: Summary Table for Population Slope (β) Estimators' Performance (Based on RMSE).

ESTIMATION METHODS	ERROR MODEL TYPE			
	SAMPLE SIZE (N=10)			
	STANDARD MODEL	**OUTLIER MODEL**	**MIXTURE MODEL**	**CONTAMINATION MODEL**
NORMAL	^{1}LSE (-), ^{2}THEIL (-0.1311) ^{3}LTS (-0.4252), ^{4}LAD (-86.4300) ^{5}wTHEIL (-435.5000)	^{1}THEIL (0.1168), ^{2}LSE (-) ^{3}LTS (-0.0195), ^{4}LAD (-143.1000) ^{5}wTHEIL (-641.7000)	^{1}LSE (-), ^{2}THEIL (-0.1573) ^{3}LTS (-0.6800), ^{4}LAD (-62.6500) ^{5}wTHEIL (-423.0000)	^{1}LSE (-), ^{2}THEIL (-0.1212) ^{3}LTS (-0.4116), ^{4}LAD (-87.3300) ^{5}wTHEIL (-436.7000)
STUDENT-T (10df)	^{1}THEIL (0.1774), ^{2}LTS (0.1489) ^{3}LSE (-), ^{4}LAD (-171.8000) ^{5}wTHEIL (-309.9000)	^{1}THEIL (0.1999), ^{2}LTS (0.1395) ^{3}LSE (-), ^{4}wTHEIL (-866.9000) ^{5}LAD (-907.1000)	^{1}THEIL (0.1761), ^{2}LTS (0.1457) ^{3}LSE (-), ^{4}LAD (-164.5000) ^{5}wTHEIL (-312.3000)	^{1}THEIL (0.2426), ^{2}LTS (0.2129) ^{3}LSE (-), ^{4}LAD (-182.5000) ^{5}wTHEIL (-317.2000)
LOGNORMAL	^{1}THEIL (0.7354), ^{2}LTS (0.5535) ^{3}LSE (-), ^{4}LAD (-16.8900) ^{5}wTHEIL (-573.7000)	^{1}THEIL (0.8002), ^{2}LTS (0.6368) ^{3}LSE (-), ^{4}LAD (-26.5800) ^{5}wTHEIL (-683.7000)	^{1}THEIL (0.6196), ^{2}LTS (0.1184) ^{3}LSE (-), ^{4}LAD (-10.4700) ^{5}wTHEIL (-508.3000)	^{1}THEIL (0.7095), ^{2}LTS (0.5177) ^{3}LSE (-), ^{4}LAD (-16.2700) ^{5}wTHEIL (-569.8000)
CAUCHY	^{1}THEIL (1.0000), ^{2}LTS (1.0000) ^{3}LAD (0.9999), ^{4}LSE (-) ^{5}wTHEIL (-1087.0)	^{1}THEIL (1.0000), ^{2}LAD (0.9999) ^{3}LTS (0.9996), ^{4}LSE (-) ^{5}wTHEIL (-803.6000)	^{1}THEIL (1.0000), ^{2}LAD (1.0000) ^{3}LTS (1.0000), ^{4}LSE (-) ^{5}wTHEIL (-1097.0000)	^{1}THEIL (0.9998), ^{2}LTS (0.9996) ^{3}LAD (0.9942), ^{4}LSE (-) ^{5}wTHEIL (-645.1000)
	SAMPLE SIZE (N=50)			
NORMAL	^{1}LSE (-), ^{2}THEIL (-0.0521) ^{3}LTS (-0.3287), ^{4}wTHEIL (-105000.0000), ^{5}LAD (NILL)	^{1}THEIL (0.5314), ^{2}LTS (0.0298) ^{3}LSE (-), ^{4}LAD (-16170.0000) ^{5}wTHEIL (-158100.00)	^{1}LSE (-), ^{2}THEIL (-0.0232) ^{3}LTS (-1.0200), ^{4}LAD (-54030.0000) ^{5}wTHEIL (-94850.0000)	^{1}THEIL (0.0276), ^{2}LSE (-) ^{3}LTS (-0.3254), ^{4}LAD (-5578.0000) ^{5}wTHEIL (-105100.0000)
STUDENT-T (10df)	^{1}THEIL (0.4962), ^{2}LTS (0.3601) ^{3}LSE (-), ^{4}wTHEIL (-107100.0000) ^{5}LAD (NILL)	^{1}THEIL (0.6123), ^{2}LTS (0.2382) ^{3}LSE (-), ^{4}wTHEIL (-231100.0000) ^{5}LAD (-501500.0000)	^{1}THEIL (0.4982), ^{2}LTS (0.3533) ^{3}LSE (-), ^{4}LAD (-60370.0000) ^{5}wTHEIL (-108100.00)	^{1}THEIL (0.6073), ^{2}LTS (0.4738) ^{3}LSE (-), ^{4}wTHEIL (-110000.0000) ^{5}LAD (-146100.00)
LOGNORMAL	^{1}THEIL (0.8430), ^{2}LTS (0.5359) ^{3}LSE (-), ^{4}LAD (-2068.0000) ^{5}wTHEIL (-252200.0000)	^{1}THEIL (0.9463), ^{2}LTS (0.6582) ^{3}LSE (-), ^{4}LAD (-2082.0000) ^{5}wTHEIL (-219100.0000)	^{1}THEIL (0.8449), ^{2}LTS (0.4391) ^{3}LSE (-), ^{4}LAD (-1160.0000) ^{5}wTHEIL (-166400.00)	^{1}THEIL (0.8388), ^{2}LTS (0.4745) ^{3}LSE (-), ^{4}LAD (-1313.0000) ^{5}wTHEIL (-245800.00)
CAUCHY	^{1}THEIL (1.0000), ^{2}LTS (1.0000) ^{3}LAD (0.9893), ^{4}LSE (-) ^{5}wTHEIL (-504200.0000)	^{1}THEIL (1.0000), ^{2}LTS (0.9998) ^{3}LAD (0.9985), ^{4}LSE (-) ^{5}wTHEIL (-295400.0000)	^{1}THEIL (1.0000), ^{2}LTS (1.0000) ^{3}LAD (0.9999), ^{4}LSE (-) ^{5}wTHEIL (-522700.0000)	^{1}THEIL (1.0000), ^{2}LTS (0.9998) ^{3}LAD (0.8352), ^{4}LSE (-) ^{5}wTHEIL (-245800.0000)
	SAMPLE SIZE (N=100)			
NORMAL	^{1}LSE (-), ^{2}THEIL (-0.0733) ^{3}LTS (-0.4427), ^{4} LAD (-78740.00) ^{5}wTHEIL (-1116000.00)	^{1}THEIL (0.6820), ^{2}LTS (0.05429) ^{3}LSE (-), ^{4} LAD (-134000.0000) ^{5}wTHEIL (-1632000.0000)	^{1}THEIL (0.0437), ^{2}LSE (-) ^{3}LTS (-1.2830), ^{4} LAD (-65360.00), ^{5}wTHEIL (-956400.00)	^{1}THEIL (0.0643), ^{2} LSE (-) ^{3} LTS (-0.4270), ^{4} LAD (-66240.00) ^{5}wTHEIL (-1111000.00)
STUDENT-T (10df)	^{1}THEIL (0.5736), ^{2}LTS (0.4182) ^{3}LSE (-), ^{4} LAD (-1030000.0000) ^{5}wTHEIL (-1332000.0000)	^{1}THEIL (0.7576), ^{2}LTS (0.2628) ^{3}LSE (-), ^{4}wTHEIL (-2385000.0000) ^{5}LAD (-6789000.0000)	^{1}THEIL (0.5729), ^{2}LTS (0.4082) ^{3}LSE (-), ^{4}LAD (-307400.0000) ^{5}wTHEIL (-1338000.0000)	^{1}THEIL (0.6968), ^{2}LTS (0.5402) ^{3}LSE (-), ^{4} LAD (-2822000.0000) ^{5}wTHEIL (-1355000.0000)
LOGNORMAL	^{1}THEIL (0.8468), ^{2}LTS (0.4297) ^{3}LSE (-), ^{4}LAD (-24670.0000) ^{5}wTHEIL (-3791000.000)	^{1}THEIL (0.9728), ^{2}LTS (0.6657) ^{3}LSE (-), ^{4} LAD (-10630.0000) ^{5}wTHWIL (-2476000.0000)	^{1}THEIL (0.8782), ^{2}LTS (0.5406) ^{3}LSE (-), ^{4}LAD (-8031.0000) ^{5}wTHEIL (-2180000.0000)	^{1}THEIL (0.8465), ^{2}LTS (0.3538) ^{3}LSE (-), ^{4} LAD (-81610.0000) ^{5}wTHEIL (-3717000.0000)
CAUCHY	^{1}THEIL (1.0000), ^{2}LTS (0.9996) ^{3}LAD (0.1219), ^{4}LSE (-) ^{5}wTHEIL (-13430000.0000)	^{1}THEIL (1.0000), ^{2}LTS (0.9989) ^{3}LAD (0.8236), ^{4}LSE (-) ^{5}wTHEIL (-5675000.0000)	^{1}THEIL (1.0000), ^{2}LTS (0.9997) ^{3}LAD (0.9775), ^{4}LSE (-) ^{5}wTHEIL (-7435000.0000)	^{1}THEIL (1.0000), ^{2}LTS (0.9997) ^{3}LAD (0.5128), ^{4}LSE (-) ^{5}wTHEIL (-16910000.0000)

Y-Intercept Estimators Performance: The performances of the population y-intercept estimators for each of the methods were found to follow the same pattern as those of the slope estimators, but for a few significant variations (Table 4). First, it was noticeable that unlike the LAD and weighted Theil's slope estimators which consistently underestimated the slope parameter β, the intercept estimators for both procedures gave a positive bias values across all cells of the simulation (Table 4). It was also observed that the variance and MSE values of LAD intercept estimators are generally consistent with those of other intercept estimators than those of its slope estimators (Tables 4 and 5). Furthermore, the variance and MSE for LSE, LTS and Theil's intercept estimators are significantly larger than those of their slope estimators, especially as sample size increases.

For the contamination model, the patterns of estimators' behavior for the intercept estimators are similar to the outliers' case. However, estimates of variance and MSE are as usual larger than those of the slope estimators.

It is worthy of note, also, that the LSE intercept estimator as long as normality assumptions hold, is by far much more efficient than every other intercept estimators regardless of sample size. But under non-normal distribution situations, Theil's, LTS and LAD take the stage as usual. Nevertheless, estimates of variance and MSE are larger and more consistent than the outliers' case for all estimators across all cells of the simulation (Tables 4, 5, 6).

CONCLUSION

This study shows that for simple linear regression model with the error terms distributed as described earlier:

- Theil estimator has high small-sample efficiency compared to the OLS estimator when the error term is heteroscedastic (Wilcox, 1998). More so, Theil's nonparametric estimation technique has the strongest performance and most reliable results and can be used in varying circumstances.

- LTS and LAD are most especially applicable when the error term comes from a heavy tailed distribution (Mutan, 2004). However, LTS proved to be more robust than LAD in most cases.

- LAD is not very robust and should be used with cautions, especially when there are uncertainties as regards the nature of the data in question.

- LSE method is only reliable As long as the normality assumption holds.

RECOMMENDATION

Application of statistical regression analysis is of great importance in the study of economy, though, most economic theories do not imply specific functional forms. It is therefore important that Statisticians and Econometricians focus more on the use as well as development of nonparametric methods and median-based estimators for econometric analyses, since it has been demonstrated that non-parametric procedures exhibit the strongest performance and most reliable results under varying data conditions.

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