Influence of Magnetic Field on Couple Stress Flow through Porous Channel with Entropy Generation

Jacob A. Gbadeyan[1] and Abiodun A. Opanuga[*2]

[1]Department of Mathematics, Faculty of Physical Science, University of Ilorin, Ilorin, Nigeria.
[2]Department of Mathematics, College of Science & Technology, Covenant University, Ota, Nigeria.

E-mail: j.agbadeyan@yahoo.com
Abiodun.opanuga@covenantuniversity.edu.ng[*]

ABSTRACT

In this article, the influence of magnetic field on couple stress flow through porous channel is considered. The flow is steady and assumed to exchange heat with the ambient based on Newtonian heating law. Since heat flows in an irreversible manner through a viscous fluid, entropy is therefore generated continuously within the flow channel. Based on this assumption, the governing equations for the flow are formulated, non-dimensionalized and the approximate solutions of the boundary value problems are obtained via the semi-analytical technique of Adomian Decomposition Method (ADM). The series solutions are validated via Differential Transform Method (DTM), the entropy generation rate and heat irreversibility are computed using the obtained ADM solution. Graphical results are presented and discussed based on the physics of the fluid.

(Keywords: magnetohydrodynamic, porous channel, entropy generation, differential transform method, DTM, Adomian decomposition method, ADM)

INTRODUCTION

Appreciable progress has been recorded in literature on factors responsible for entropy generation due the increased drive to minimize wastage of scarce resources. Noticeable among these studies include: Bouabid et al. [1] in which entropy generation by free convection in an inclined rectangular channel was investigated.

Adesanya et al. [2] discussed the effects of couple stresses and Navier slip on the entropy generation. Also, Jery et al. [3] studied the effect of inclined magnetohydrodynamics (MHD) on the rate of entropy generation by considering only the natural convention flow. Basak et al. [4] considered the effect of thermal boundary conditions on the entropy generation for free convection with Darcian porous cavity.

Furthermore, Opanuga et al. [5] investigated the effect of thermal radiation on the entropy generation of hydromagnetic flow through porous channel, it was stated that entropy generation rises with the increase in the radiation parameter. Also Opanuga et al. [6] considered second law analysis of a reactive MHD couple stress fluid through porous channel, the study showed that increase in magnetic field and Frank-Kameneskii parameters increases the entropy generation. Other important studies on entropy generation analysis in a moving fluid under diverse geometry can be found in [7-14].

Investigations have been geared towards the studies of MHD both as a flow control techniques and to agglomerate flow particles in many industrial processes. Some of these findings were documented in recent work in the literature for instance; Turkyilmazoglu [15] documented some findings on the existence and limit of heat and mass transfer of hydromagnetic slip flow for viscous fluid. Also Fang et al. [16] obtained the exact solution for slip magnetohydrodynamic viscous fluid. Similarly, Beg et al. [17] presented the study of free convective hydrodynamic heat and mass transfer between a stretching surface and highly porous medium with soret and Dufour effects. Adesanya et al. [18] considered the entropy generation associated with MHD third grade fluid through porous medium. Other related work on MHD flow can be found in [19-24].

Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) applied in this work have been noted for their high accuracy, rapid convergence and simplicity in handling boundary value problems, see Refs. [25-26].

In the present work, emphasis is on the control of heat irreversibility which could arise from viscous dissipation or heat transfer. This is because heat irreversibility is generated continuously with temperature difference across the heated channel. This all-important effect has been neglected in the recent work by Adesanya and Makinde in spite of various MHD applications in industrial and engineering systems involving heat transfer. In the following section, the mathematical model for the thermal analysis is formulated, non-dimensionalized and solved, graphical results are presented, and the work is concluded.

MODEL FORMULATION

We consider the steady flow and heat transfer of hydromagnetic couple stress fluid flowing steadily through a porous channel taking Ohmic heating of the fluid into consideration. A magnetic field of strength B_0 is applied perpendicularly to the channel plates. It is assumed that the exchange of heat with the ambient is in axi-symmetrical manner. Fluid is injected with a constant velocity v_0 at the injection wall $y=0$ while it is sucked off at the upper wall $y=h$ with the same velocity.

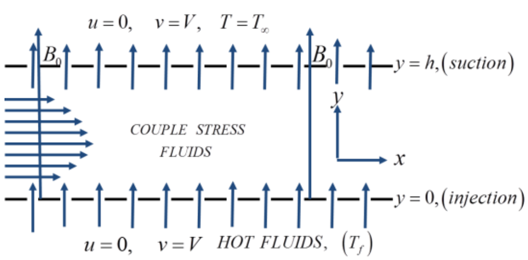

Figure 1: Schematic Diagram of the Problem.

Under this configuration (see Figure 1), the governing equations for the flow can be written as follows Adesanya et al. [5].

$$\rho v_\circ \frac{du'}{dy'} = -\frac{dp}{dx'} + \mu \frac{d^2u'}{dy'^2} - \eta \frac{d^4u'}{dy'^4} - \sigma B_\circ^2 u' \quad \textbf{(1)}$$

$$\rho c_p v_0 \frac{dT'}{dy'} = k\frac{d^2T'}{dy'^2} + \mu\left(\frac{du'}{dy'}\right)^2 + \eta\left(\frac{d^2u'}{dy'^2}\right)^2 + \sigma B_\circ^2 u'^2 \quad \textbf{(2)}$$

In Equations (1-2) the last terms are due to the effect of magnetic field arising from interactions with the fluid. The thermal boundary conditions are:

$$k\frac{dT'}{dy'}(0) = -\gamma_1(T_f - T'), \quad k\frac{dT'}{dy'}(h) = -\gamma_2(T' - T_\circ) \quad \textbf{(3)}$$

while the no-slip and the stress-free conditions are given as:

$$u'(0) = \frac{d^2u'}{dy^2}(0) = 0 = u'(h) = \frac{d^2u'}{dy^2}(h) \quad \textbf{(4)}$$

The dimensionless parameters for this flow are:

$$y = \frac{y^I}{h}, u = \frac{u^I}{v_\circ}, \theta = \frac{T^I - T_\circ}{T_f - T_\circ}, s = \frac{v_\circ h}{\nu},$$
$$G = \frac{-h^2}{\mu v_\circ}\frac{dp}{dx}, a^2 = \mu\frac{h^2}{\eta}, \Pr = \frac{\nu\rho c_p}{k},$$
$$Br = \frac{\mu v_\circ}{k(T_f - T_\circ)}, Bi_1 = \frac{\gamma_1 h}{k}, Bi_2 = \frac{\gamma_2 h}{k}, \quad \textbf{(5)}$$
$$Ns = \frac{T_\circ^2 h^2 E_G}{k(T_f - T_\circ)^2}, \Omega = \frac{T_f - T_\circ}{T_\circ}, \nu = \frac{\mu}{\rho},$$
$$H^2 = \frac{\sigma B_0^2 h^2}{\mu}$$

Using Equation (5) in Equations (1-4) yields the boundary-value problems:

$$s\frac{du}{dy}=G+\frac{d^2u}{dy^2}-\frac{1}{a^2}\frac{d^4u}{dy^4}-H^2u,$$
$$u(0)=\frac{d^2u}{dy^2}(0)=0=u(1)=\frac{d^2u}{dy^2}(1) \quad \textbf{(6)}$$

$$\frac{d^2\theta}{dy^2}=sp_r\frac{d\theta}{dy}-Br\left\{\begin{matrix}\left(\frac{du}{dy}\right)^2\\ +\frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2+H^2u^2\end{matrix}\right\}, \quad \textbf{(7)}$$
$$\frac{d\theta}{dy}(0)=Bi_1(\theta(0)-1),\ \frac{d\theta}{dy}(1)=-Bi_2\theta(1)$$

The skin friction coefficient and the rate of heat transfer at the channel walls respectively are given as:

$$C_f=\left.\frac{du(y)}{dy}\right|_{y=0,1};Nu=-\left.\frac{d\theta(y)}{dy}\right|_{y=0,1} \quad \textbf{(8)}$$

METHOD OF SOLUTION

Writing Equations (6)-(7) in the integral forms, we obtain:

$$u(y)=b_1y+\frac{b_2}{3!}y^3+ a^2\int_0^y\int_0^y\int_0^y\int_0^y\left\{\begin{matrix}G+\frac{d^2u}{dY^2}-\\ s\frac{du}{dY}-H^2u\end{matrix}\right\}dYdYdYdY \quad \textbf{(9)}$$

and

$$\theta(y)=b_3+b_4y+ \int_0^y\int_0^y\left\{\begin{matrix}s\Pr\frac{d\theta}{dY}-Br\left(\frac{du}{dY}\right)^2\\ -\frac{Br}{a^2}\left(\frac{d^2u}{dY^2}\right)^2-BrH^2u^2\end{matrix}\right\}dYdY \quad \textbf{(10)}$$

where, b_1, b_2, b_3, b_4 are the parameters to be determined later.

By ADM, we define an infinite series solution of the form:

$$u(y)=\sum_{n=0}^{\infty}u_n(y),\ \theta(y)=\sum_{n=0}^{\infty}\theta_n(y), \quad \textbf{(11)}$$

Now using Equation (11) in (9)-(10), we have:

$$\sum_{n=0}^{\infty}u_n(y)=b_1y+\frac{b_2}{3!}y^3+ a^2\int_0^y\int_0^y\int_0^y\int_0^y\left\{\begin{matrix}G+\frac{d^2u}{dY^2}-\\ s\frac{du}{dY}+H^2U\end{matrix}\right\}dYdYdYdY \quad \textbf{(12)}$$

and

$$\sum_{n=0}^{\infty}\theta(y)=b_3+b_4y+ \int_0^y\int_0^y\left\{\begin{matrix}s\Pr\frac{d\theta}{dY}-Br\left(\frac{du}{dY}\right)^2-\\ \frac{Br}{a^2}\left(\frac{d^2u}{dY^2}\right)^2+BrH^2u^2\end{matrix}\right\}dYdY \quad \textbf{(13)}$$

In view of Equations (12)-(13), the zeroth order term can be written as:

$$\sum_{n=0}^{\infty}u_n(y)= b_1y+\frac{b_2}{3!}y^3+a^2\int_0^y\int_0^y\int_0^y\int_0^y\{G\}dYdYdYdY \quad \textbf{(14)}$$

and

$$\sum_{n=0}^{\infty}\theta(y)=b_3+b_4y \quad \textbf{(15)}$$

while other terms can be determined using the recurrence relations:

$$\sum_{n=0}^{\infty} u_n(y) =$$
$$a^2 \int_0^y\int_0^y\int_0^y\int_0^y \left\{ \frac{d^2u}{dY^2} - s\frac{du}{dY} + H^2U \right\} dYdYdYdY \quad \textbf{(16)}$$

and

$$\sum_{n=0}^{\infty} \theta(y) =$$
$$\int_0^y\int_0^y \left\{ \begin{array}{l} s\Pr\dfrac{d\theta}{dY} - Br\left(\dfrac{du}{dY}\right)^2 - \\ \dfrac{Br}{a^2}\left(\dfrac{d^2u}{dY^2}\right)^2 - BrH^2u^2 \end{array} \right\} dYdY \quad \textbf{(17)}$$

The calculations associated with the integral Equations (14)-(17) are done by coding (14)-(17) in an algebra symbolic package-Mathematica. Only the graphical results are presented in Figures (2)-(13) due to the size of the approximate solution. The accuracy of these computations is verified by comparing the approximate solution obtained via ADM with the DTM and the exact solutions in Table 1.

Table 1: Computation showing convergence of solution when $G = H = a = 1, s = 0.1$

y	EXACT SOLUTION	DTM ERROR	ADM ERROR
0	0	0	0
0.1	0.003680317	7.64715E-13	2.2353E-12
0.2	0.006959623	1.66769E-12	4.3323E-12
0.3	0.009523962	2.83267E-12	6.1673E-12
0.4	0.011151328	4.36902E-12	7.631E-12
0.5	0.011709275	6.3757E-12	8.6243E-12
0.6	0.011153527	8.94183E-12	9.0581E-12
0.7	0.009527596	1.2152E-11	8.8479E-12
0.8	0.006963372	1.60877E-11	7.912E-12
0.9	0.003682714	2.08323E-11	6.167E-12
1	-7.00E-17	2.64749E-11	3.5233E-12

Entropy Generation Analysis

The local entropy generation rate Bejan [25] for the problem under discussion can be written as:

$$E_G = \frac{k}{T_\circ^2}\left(\frac{dT'}{dy'}\right)^2 + \frac{\mu}{T_\circ}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_\circ}\left(\frac{d^2u'}{dy'^2}\right)^2 + \frac{\sigma B_\circ^2 u'^2}{T_\circ} \quad \textbf{(18)}$$

so that with (3), the dimensionless form becomes:

$$Ns = \left(\frac{d\theta}{dy}\right)^2 + \frac{Br}{\Omega}\left\{ \begin{array}{l} \left(\dfrac{du}{dy}\right)^2 + \\ \dfrac{1}{a^2}\left(\dfrac{d^2u}{dy^2}\right)^2 + H^2u^2 \end{array} \right\} \quad \textbf{(19)}$$

$\left(\dfrac{d\theta}{dy}\right)^2$ is the irreversibility due to heat transfer,

$\dfrac{Br}{\Omega}\left(\dfrac{du}{dy}\right)^2$ is entropy generation due to fluid friction,

while the last two terms are couple stresses and magnetic field irreversibility respectively. Setting, N_1 as the irreversibility due to heat transfer and N_2 as hydromagnetic fluid friction irreversibility.

$$N_1 = \left(\frac{d\theta}{dy}\right)^2,$$
$$N_2 = \frac{Br}{\Omega}\left\{ \left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2 + H^2u^2 \right\} \quad \textbf{(20)}$$

then the Bejan number (Be) for the irreversibility ratio can be written as:

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\Phi}, \quad \Phi = \frac{N_2}{N_1} \quad \textbf{(21)}$$

Observe Be = 0 corresponds to the special case when heat irreversibility due to magnetic fluid friction dominates over heat transfer irreversibility. Similarly, as N_1 takes preeminence over N_2 the numerical value of Be is expected to be unity and ½ when both contributes equally.

RESULTS AND DISCUSSION

In this study, the influence of hydromagnetic couple stress fluid with entropy generation through porous channel is analyzed. Semi-analytical solutions via Adomian decomposition and differential transformation methods for the velocity and temperature profiles obtained are used for entropy generation rate. To be realistic values for Prandtl number are chosen between 0.004 and 2 to include fluids such as mercury (0.008-0.041), oxygen (0.729-0.759), air (0.703-0.784) and water vapor (0.882-0.994) [23].

Effect of Parameters Variation on Velocity and Temperature Profiles

The effects of parameters variation on velocity and temperature profiles are displayed in Figures 2-7. Figure 2 shows the effect of magnetic field parameter on fluid velocity. It is observed that an increase in the magnetic field parameter (H^2) retards fluid velocity. This is as a result of the presence of Lorentz force that opposes the fluid flow; the force is caused by the applied magnetic field which clusters the fluid particles and thus impedes the free flow of the fluid.

In Figure 3 variation in suction/injection parameter on fluid velocity is presented. It is observed that as injection of hot fluid into the channel increases fluid velocity and leads to a shift in flow symmetry.

Figure 4 illustrates the effect of couple stress inverse parameter on fluid velocity. Increase in the inverse of couple stresses increases fluid velocity which indicates that couple stress parameter will decrease fluid velocity. The result is physically true because increase in couple stresses enhances fluid viscosity as fluid particles size increases. This confirms the result reported in Adesanya et al. [5].

Furthermore, Figure 5 depicts the effect of magnetic field on fluid temperature. As seen from the plot, there is an increase in fluid temperature as magnetic field parameter (H^2) increases. The increase can be attributed to the increase in viscous heating which increases the rate of heat transfer from the fluid to the walls.

Figure 6 shows the plot of suction/injection on temperature profile. Expectedly, an increase in temperature is noted as injection of hot fluid into the channel increases.

Figure 7 presents the effect of couple stress inverse on temperature. It shows that fluid temperature is reduced as couple stress parameter increases. The reason is that fluid viscosity rises with increase in couple stresses.

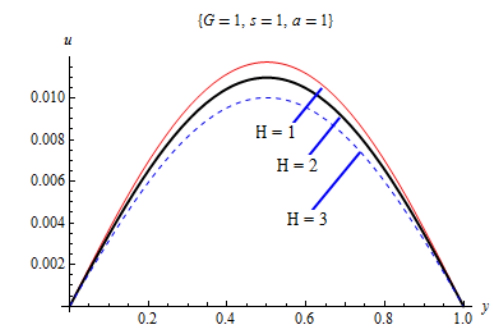

Figure 2: Effect of Magnetic Field Parameter on Fluid Velocity.

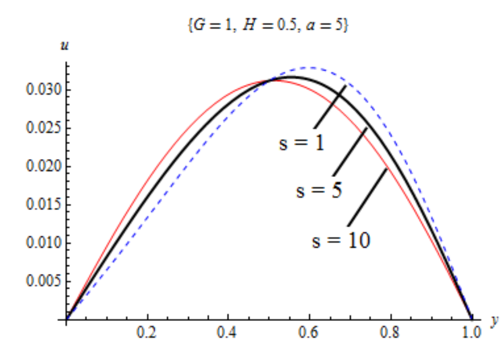

Figure 3: Effect of Suction/Injection Parameter on Fluid Velocity.

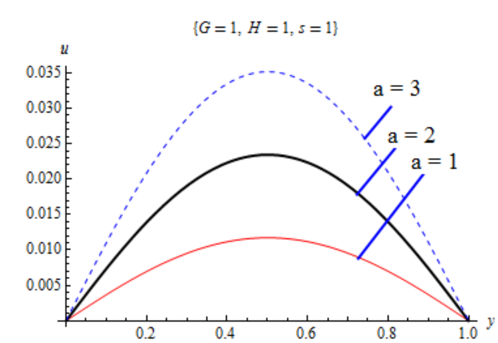

Figure 4: Effect of Couple Stress Inverse on Fluid Velocity.

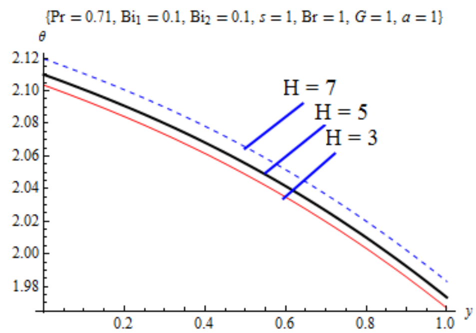

Figure 5: Effect of Couple Stress Inverse on Fluid Temperature.

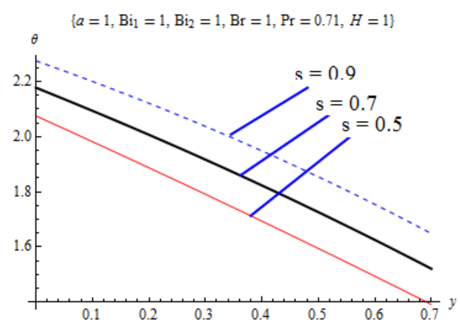

Figure 6: Effect of Suction/Injection Parameter on Fluid Velocity.

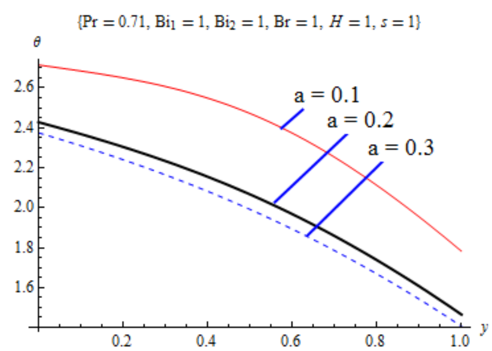

Figure 7: Effect of Couple Stress Inverse Parameter on Fluid Temperature.

Effect of Parameters Variation on Entropy Generation Rate

In Figures 8-10, the effects of variation of some governing parameters on entropy generation rate are presented. Figure 8 depicts the plot of variation in magnetic field parameter on entropy generation.

As shown in Figure 2, the rise in Hartman number decreases the flow velocity while increasing the fluid temperature as reported in Figure 5. The net effect on entropy generation rate is presented here in Figure 8. As observed, increased Hartman number enhances entropy generation. This is true since Hartman number decreases velocity thereby clustering the fluid particle. This clustering enhances viscous dissipation within the layers. Moreover, Hartman number increases fluid temperature implies that the rate of heat transfer from the fluid to the walls must increase. The totality of these is shown to increase entropy generation significantly as confirmed in the plot.

In Figure 9, it is shown that entropy generation decreases at the lower wall with injection and in the middle of the channel while it increases at the upper wall with suction. This can be traced to Figure 3 where it is depicted that increase in suction/injection parameter leads to a break in flow symmetry within the channel.

Figure 10 depicts the effect of couple stresses on the entropy generation rate, it is observed that entropy generation increases with increase in couple stress inverse. This implies that increase in couple stresses decreases entropy generation rate. The reason is show in Figure 4, the plot indicates that fluid velocity reduces with increase in couple stresses due to the rise in fluid viscosity as fluid particle size increases. The effect of this on the flow system is to lower the random movement of the fluid particles and consequently the drop in entropy production rate more in the core region of the channel than the channel walls.

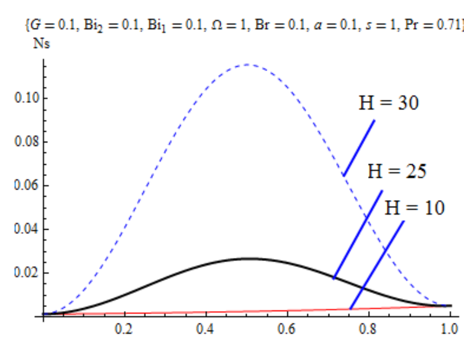

Figure 8: Effect of Magnetic Parameter Field on Entropy Generation Rate.

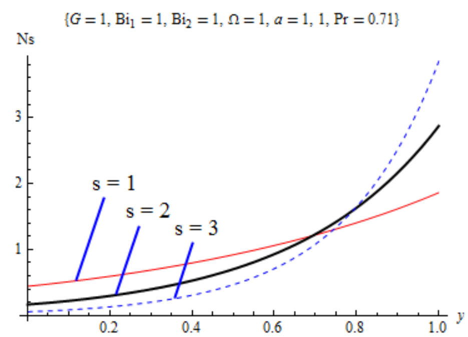

Figure 9: Effect of Suction/Injection on Entropy on Entropy Generation Rate.

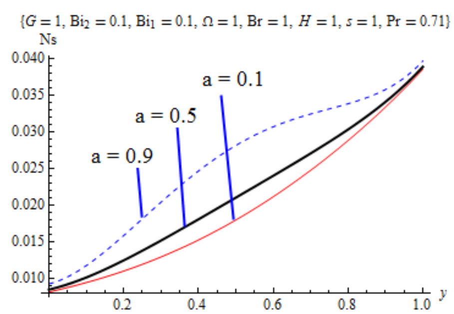

Figure 10: Effect of Couple Stress Inverse on Entropy Generation Rate.

Effect of Parameters Variation on Bejan Number

The influence of some parameters on Bejan number is presented in Figures 11-13. Figure 11 depicts the graph of magnetic field parameter on Bejan number. It is observed that Bejan number increases across the channel as magnetic field parameter increases. This is an indication that heat transfer is the major contributor to irreversibility across the channel.

Figure 12 represents the effect of suction/injection on Bejan number. The plot indicates that increase in suction/injection decreases Bejan number at the lower wall and at the center of the channel with a slight increase at the upper wall. This shows that irreversibility due to viscous dissipation dominates the flow at the upper wall.

Finally the effect of couple stress inverse on entropy generation in Figure 13 is presented; it is found that couple stress inverse increases Bejan number, which indicates that couple stresses retard Bejan number. This suggests the dominance of irreversibility by viscous dissipation over heat transfer..

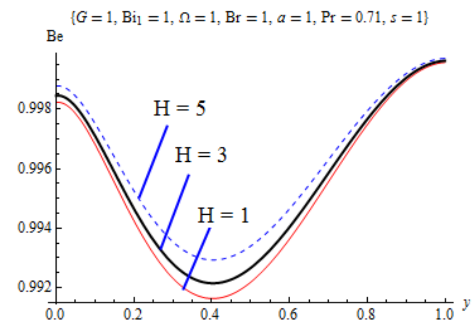

Figure 11: Effect of Magnetic Field on Bejan Entropy Generation Rate.

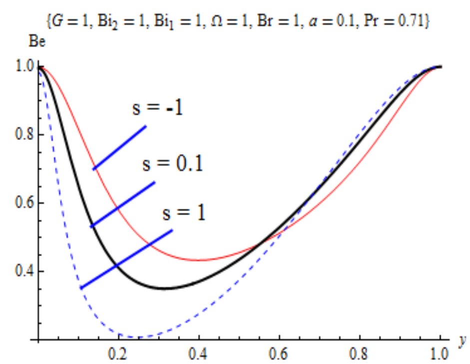

Figure 12: Effect of Suction/Injection on Bejan.

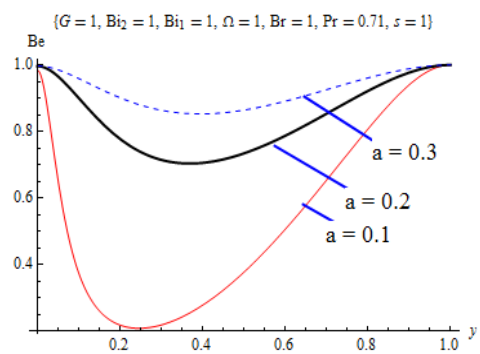

Figure 13: Effect of Couple Stress Inverse Parameter Number.

Variation of Parameters Effect on on Skin Friction

Tables 3-5 present the effect of parameters variation on skin friction coefficient. Tables 3 and 4 indicate that skin friction decreases as the value of magnetic field parameter and suction/injection increase while in Table 5 skin friction increases with increase in pressure gradient parameter.

Table 3: Effect of Magnetic Field Variation on Skin Friction.

G	H	S	a	Cf
1	1	1	1	0.03535583
1	2	1	1	0.0344646
1	3	1	1	0.0331398

Table 4: Effect of Suction/Injection on Variation on Skin Friction.

G	H	s	a	Cf
1	1	1	1	0.03535583
1	1	2	1	0.0352547
1	1	3	1	0.0351476

Table 5: Effect of Pressure Gradient Variation on Skin Friction.

G	H	s	a	Cf
1	1	1	1	0.0353583
2	1	1	1	0.0707166
3	1	1	1	0.106075

Variation of Parameters Effect on Nusselt Number

Tables 6-8 present the effect of parameters variation on Nusselt number. Table 6 presents the effect of variation in magnetic field parameter on Nusselt number. It is noticed that increase in magnetic field parameter leads to a decrease in Nusselt number. Table 7 illustrates suction/injection variation on Nusselt number. Increase in suction/injection decreases Nusselt number. Table 8 displays the effect of couple stresses on Nusselt number. The table shows that increase in couple stresses increases the Nusselt number.

Table 6: Effect of Magnetic Field Variation on Nusselt Number.

G	H	s	Pr	Br	A	Nu
1	1	1	0.71	1	1	0.0605638
1	2	1	0.71	1	1	0.0605517
1	3	1	0.71	1	1	0.605316

Table 7: Effect of Suction/Injection Variation on Nusselt Number.

G	H	s	Pr	Br	A	Nu
1	1	1	0.71	1	1	0.605638
1	1	2	0.71	1	1	0.312725
1	1	3	0.71	1	1	0.144648

Table 8: Effect of Couple Stress Variation on Nusselt Number.

G	H	s	Pr	Br	A	Nu
1	1	1	0.71	1	1	0.605638
1	1	1	0.71	1	2	0.608578
1	1	1	0.71	1	3	0.609122

CONCLUSION

In this work, entropy generation of hydromagnetic couple stress fluid in a porous channel is studied by semi-analytical techniques, DTM and ADM. The study reveals among other things that:

i. Fluid velocity decreases as magnetic field and couple stress inverse parameters increase,

ii. Fluid temperature rises with increase in magnetic field, suction/injection and couple stress inverse parameters,

iii. Entropy generation rate increases with increase in magnetic field parameter while couple stress parameter lowers entropy generation rate,

iv. Increase in magnetic field parameter increases Bejan number while increase in couple stresses retards Bejan number. However, suction/injection decreases the Bejan number at lower wall and centre of the channel

v. Irreversibility due to heat transfer dominates at the upper wall,

vi. Skin friction decreases with increase in magnetic field and suction/injection parameters. There is a decrease in the Nusselt number with increase in magnetic field and suction/injection parameters

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ABOUT THE AUTHORS

Prof. J.A. Gbadeyan, is a Professor in the Department of Mathematics, University of Ilorin, Ilorin, Nigeria. +2348035785943 j.agbadeyan@yahoo.com

Dr. A.A. Opanuga, is a Lecturer in the Department of Mathematics, Covenant University, Ota Nigeria, +2348069841259, abiodun.opanuga@covenantuniversity.edu.ng

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