Reduction of Simple Semi-Conditional Grammars.

R.O. Oladele, Ph.D. and S.N. Isah, M.Sc.

Department of Computer Science, University of Ilorin, P.M.B. 1515, Ilorin, Nigeria.

E-mail: roladele@unilorin.edu.ng
14-52he017pg@students.unilorin.edu.ng

ABSTRACT

A study on the descriptional complexity of simple semi-conditional grammars is presented. It is proved that every recursively enumerable language can be generated by a simple semi-conditional grammar of degree (2, 1) with no more than six conditional productions and seven non-terminals.

(Keywords: descriptional complexity, simple semi-conditional grammar, formal languages)

INTRODUCTION

Descriptional complexity aspects of systems (automata, grammars, rewriting systems, etc.) have been a subject of intensive research since the beginning of computer science, but the field has also been actively studied in recent years. Examples of some early results appeared in [4] about the size of context-free grammars, and the size measures of the number of nonterminal symbols and the number of productions were also introduced, see also [6] for a survey of "grammatical complexity" of context-free grammars. The succinctness of representations of languages by different variants of automata were also considered by several authors, see [3] and [5] for more details, and a survey of results in these areas.

It is obvious that the fact that a system is able to simulate some universal device implies that its size parameters can be bounded. This holds, since by simulating the universal device, all computations are carried out by a fixed (universal) system (having, therefore, fixed size parameters). On the other hand, it is still interesting to look for the best possible values of the bounds, or to study the relationship of certain size parameters with each other or with other properties of the given system.

Simple semi-conditional grammars represent a straightforward simplification of semi-conditional grammars, in which each rule has at most one nonempty condition, that is, no controlling context condition at all, or either a permitting, or a forbidding context. Semi-conditional grammars are context-free grammars, in which both a permitting context and a forbidding context are associated with each production rule.

PRELIMINARIES AND DEFINITIONS

This paper assumes that the reader is familiar with the language theory (see [7, 8]). Let V be an alphabet. V^* denotes the free monoid generated by V under the operation of concatenation where ε denotes the unit of V^*. Let $V^+ = V^* - \{ \varepsilon \}$. Given a word, $w \in V^*$, $|w|$ represents the length of w. We set sub(w) = {y : y is a subword of w}. Given a symbol, $a \in V$, $\#_a w$ denotes the number of occurrences of a in w. For $w \in V^+$, first (w) denotes the leftmost symbol of w.

A semi-conditional grammar (as n sc-grammar for short) is a quadruple, G = (V, T, P, S), where V, T and S are the total alphabet, the terminal alphabet ($T \subset V$), and the axiom ($S \in V–T$), respectively, and P is a finite set of productions of the form $(A \rightarrow x, \alpha, \beta)$ with $A \in V–T$, $x \in V^*$, $\alpha \in V^+ \cup \{0\}$ and $\beta \in V^+ \cup \{0\}$, where 0 is a special symbol, $0 \notin V$ (intuitively, 0 means that the production's condition is missing).

Production $(A \rightarrow x, \alpha, \beta) \in P$ is said to be conditional, if $\alpha \neq 0$ or $\beta \neq 0$. G has degree (i, j), where i and j are two natural numbers, if for every $(A \rightarrow x, \alpha, \beta) \in P$, $\alpha \in V^+$ implies $|\alpha| \leq i$, and $\beta \in V^+$ implies $|\beta| \leq j$. Let $u, v \in V^*$, and $(A \rightarrow x, \alpha, \beta)$

$\in$ P. Then, u directly derives v according to (A $\to$ x, α, β) in G, denoted by:

$u \Rightarrow_G v$ [(A $\to$ x, α, β)]

provided for some u_1, $u_2 \in V^*$, the following conditions (a) through (d) hold:

(a) $u = u_1Au_2$,
(b) $v = u_1xu_2$,
(c) $\alpha \neq 0$ implies $\alpha \in sub(u)$
(d) $\beta \neq 0$ implies $\beta \notin sub(u)$

When no confusion exists, we simply write $u \Rightarrow_G v$. As usual, we extend $\Rightarrow_G$ to $\Rightarrow^i_G$ (where i ≥ 0), and $\Rightarrow^*_G$. The language of G, denoted by L(G), is defined as L(G) = w $\in T^*$: S $\Rightarrow^*_G$ w.

Based upon the concept of sc-grammars, Meduna and Gopalaratnam [2] have defined as a simple semi-conditional grammar (an ssc-grammar for short) as an sc-grammar in which every production has no more than one condition. Formally, let G = (V, T, P, S) be an sc-grammar. G is a simple semi-conditional grammar if (A $\to$ x, α, β) $\in$ P implies $\{0\} \subseteq \{\alpha, \beta\}$

MAIN RESULTS

Theorem 1. Every recursively enumerable language is generated by a simple semi-conditional grammar of degree (2, 1) with no more than 6 conditional productions and 7 non-terminals.

Proof. Let L be a recursively enumerable language. From [1], we can assume that L is generated by a grammar G of the form

$$G = (V, T, P \cup \{AA \to \varepsilon, BBB \to \varepsilon\}, S)$$

such that P contains only context-free productions and

V - T = {S, A, B}

We construct an simple semi-conditional grammar G' of degree (2, 1) as follows:

G' = (V', T, P', S),

where

V' = V U W

W = {A', B', \$, #}, $V \cap W = \emptyset$

The set of productions P is defined in the following way:

1. If $H \to \alpha \in P$, $H \in V - T$, $\alpha \in V^*$, then add (H $\to$ α,0,0) to P';

2. Add the following three productions to P':

(A $\to$ A', 0, A')

(A' $\to$ \$, 0, \$)

(\$$\to$ ε, 0, A')

3. Add the following three productions to P':

(B $\to$ #B', 0, B')

(B' $\to$ ε, 0, #)

(#$\to$ ε, #B', 0)

Next we prove that L (G') = L (G).

Basic Idea: Note that G' is of degree (2, 1) and has only 6 conditional productions. The productions of (2) simulate the application of AA $\to \varepsilon$ in G' and the productions of (3) simulate the application of BBB $\to$ ε in G'.

The simulation of AA $\to \varepsilon$ is described as follows: the first occurrence of A is rewritten as A', then A' is rewritten as \$.

Finally \$ is erased. The second occurrence of A is erased in a similar version. BBB $\to$ ε is simulated in a similar way using production (4) (6) (5) (4) (6) (5) (4) (6) (5).

To establish L(G) = L(G') formally, we first prove the following claim.

Claim 1. $S \Rightarrow^* G'$ x' implies $\#_{\bar{x}} x' \leq 1$ for each $\bar{x} \in$ {A', B'}, where $x' \in (V')^*$.

Proof. By inspection of productions in P', the only productions that can generate $\bar{x}$ are of the form (x $\to$ $\bar{x}$, 0, $\bar{x}$) or (x $\to$ $\#\bar{x}$, 0, $\bar{x}$). These productions can be applied only when no $\bar{x}$ occurs in the rewritten sentential form. Thus, it is impossible to derive x' from S such that $\#_{\bar{x}} x' \geq 2$.

Let g be a finite substitution from $(V')^*$ to V^* defined as follows:

1. for all $X \in V$: $g(X) = \{X\}$;
2. $g(A') = \{A\}$, $g(\$) = \{A, AA\}$;
3. $g(B') = \{\varepsilon, B, BBB\}$, $g(\#) = \{B, BBB\}$.

Claim 2. $S \Rightarrow^*_G x$ if and only if $S \Rightarrow^*_{G'} x'$ for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Proof. This claim is proved by induction on the length of derivations. Only if, we prove that

$S \Rightarrow^m_G x$ implies $S \Rightarrow^*_{G'} x$,

where $m \geq 0$, $x \in V^*$. This is established by induction on m.

Basis: Let m = 0. That is $S \Rightarrow^0_G S$. Clearly, $S \Rightarrow^0_{G'} S$.

Induction Hypothesis: Suppose that the claim holds for all derivations of length m or less for some $m \geq 0$.

Induction Step: Let us consider a derivation $S \Rightarrow_G^{m+1} x$, $x \in V^*$. Since $m + 1 \geq 1$, there is some $y \in V^+$ and $p \in P \cup \{AA \rightarrow \varepsilon, BBB \rightarrow \varepsilon\}$ such that $S \Rightarrow^m_G y \Rightarrow_G x$ [p]. By the induction hypothesis there is a derivation $S \Rightarrow^*_{G'} y$. There are three cases that cover all possible forms of the production p:

(i) $p = H \rightarrow y_2 \in P$, $H \in V - T$, $y_2 \in V^*$.
Then $y = y_1Hy_3$ and $x = y_1y_2y_3$,
$y_1, y_3 \in V^*$. Because we have $(H \rightarrow y_2, 0, 0) \in P'$, $S \Rightarrow^*_{G'} y_1Hy_3 \Rightarrow_{G'} y_1y_2y_3$ $[(H \rightarrow y_2, 0, 0)]$ and $y_1y_2y_3 = x$.

(ii) $p = AA \rightarrow \varepsilon$. Then $y = y_1AAy_3$ and $x = y_1y_3$, $y_1, y_3 \in V^*$. In this case there is the following derivation which use the production:

$S \Rightarrow^*_{G'}\ y_1AAy_3$
${}^1\Rightarrow^*_{G'}\ y_1A'Ay_3\ [(A \rightarrow A', 0, A')]$
${}^2\Rightarrow^*_{G'}\ y_1\$Ay_3\ [(A' \rightarrow \$, 0, \$)]$
${}^3\Rightarrow^*_{G'}\ y_1Ay_3\ [(\$ \rightarrow \varepsilon, 0, A')]$
${}^1\Rightarrow^*_{G'}\ y_1A'y_3\ [(A \rightarrow A', 0, A')]$
${}^2\Rightarrow^*_{G'}\ y_1\$y_3\ [(A' \rightarrow \$, 0, \$)]$
${}^3\Rightarrow^*_{G'}\ y_1y_3\ [(\$ \rightarrow \varepsilon, 0, A')]$

(iii) $p = BBB \rightarrow \varepsilon$. Then $y = y_1BBBy_3$ and $x = y_1y_3$, $y_1, y_3 \in V^*$. In this case there exist the derivation:

$S \Rightarrow^*_{G'}\ y_1BBBy_3$
${}^4\Rightarrow^*_{G'}\ y_1BB\#B'y_3\ [(B \rightarrow \#B', 0, B')]$
${}^6\Rightarrow^*_{G'}\ y_1BBB'y_3\ [(\#B' \rightarrow \varepsilon, 0, \$)]$
${}^5\Rightarrow^*_{G'}\ y_1BBy_3\ [(B' \rightarrow \varepsilon, 0, \#)]$
${}^4\Rightarrow^*_{G'}\ y_1B\#B'y_3\ [(B \rightarrow \#B', 0, B')]$
${}^6\Rightarrow^*_{G'}\ y_1BB'y_3\ [(\#B' \rightarrow \varepsilon, 0, \$)]$
${}^5\Rightarrow^*_{G'}\ y_1By_3\ [(B' \rightarrow \varepsilon, 0, \#)]$
${}^4\Rightarrow^*_{G'}\ y_1\#B'y_3\ [(B \rightarrow \#B', 0, B')]$
${}^6\Rightarrow^*_{G'}\ y_1B'y_3\ [(\#B' \rightarrow \varepsilon, 0, \$)]$
${}^5\Rightarrow^*_{G'}\ y_1y_3\ [(B' \rightarrow \varepsilon, 0, \#)]$

If : By induction on $n \geq 0$, we prove that

$S \Rightarrow^n_{G'} x'$ implies $S \Rightarrow^*_G x$

for some $x \in g(x')$, $x \in V^*$, $x' \in (V')^*$.

Basis: Let n = 0. That is, $S \Rightarrow^0_{G'} S$. It is obvious that $S \Rightarrow^0_G S$ and $S \in g(S)$.

Induction Hypothesis: Assume that the claim holds for all derivations of length n or less, for some $n \geq 0$.

Induction Step: Consider a derivation $S \Rightarrow_{G'}^{n+1} x'$, $x' \in (V')^*$. Since $n + 1 \geq 1$, there is some $y' \in (V')^+$ and $p' \in P'$ such that $S \Rightarrow^n_{G'} y' \Rightarrow_{G'} x'$ [p'] and by the induction hypothesis there is also a derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$. By inspection of P' the following cases (i) through (xvi) cover all possible forms of p':

(i) $p' = (H \rightarrow y_2, 0, 0) \in P'$, $H \in V - T$, $y_2 \in V^*$. Then $y' = y'_1Hy'_3$, $x' = y'_1y_2y'_3$, $y'_1, y'_3 \in (V')^*$ and y has the form $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(H)$. Because for all $X \in V - T$ such that $g(X) = X$, the only Z is H and thus $y = y_1Hy_3$. By the definition of P' (see (1)) there exists a production $p = H \rightarrow y_2$ in P and we can construct the derivation $S \Rightarrow^*_G y_1Hy_3 \Rightarrow_G y_1y_2y_3$ [p] such that $y_1y_2y_3 = x$, $x \in g(x')$.

(ii) $p' = (A \rightarrow A', 0, A')$. Then $y' = y'_1Ay'_3$, $x' = y'_1A'y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(A)$. Because $g(A) = \{A\}$ the only Z is A, so we can express $y = y_1Ay_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $A \in$

g(A')

(iii) p' = (A' → \$, 0, \$). Then $y' = y'_1A'y'_3$, $x' = y'_1\$y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(A')$. Because g(A') = {A} the only Z is A, so we can express $y = y_1Ay_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $A \in g(\$)$

(iv) p' = (\$→ ε, 0, A'). Then, $y' = y'_1\$y'_3$ and $x' = y'_1y'_3$, where $y'_1, y'_3 \in (V')^*$. Express $y = y_1Zy_3$ so that $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(\$)$, where g(\$) = {A, AA}. Let Z = AA. Then, $y = y_1AAy_3$ and there exists the derivation $S \Rightarrow^*_G y_1AAy_3 \Rightarrow_G y_1y_3$ [AA → ε], where $y_1y_3 = x$, $x \in g(x')$.

(v) p' = (A → A', 0, A'). Then $y' = y'_1Ay'_3$, $x' = y'_1A'y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(A)$. Because g(A) = {A} the only Z is A, so we can express $y = y_1Ay_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $A \in g(A')$

(vi) p' = (A' → \$, 0, \$). Then $y' = y'_1A'y'_3$, $x' = y'_1\$y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(A')$. Because g(A') = {A} the only Z is A, so we can express $y = y_1Ay_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $A \in g(\$)$

(vii) p' = (\$→ ε, 0, A'). Then, $y' = y'_1\$y'_3$ and $x' = y'_1y'_3$, where $y'_1, y'_3 \in (V')^*$. Express $y = y_1Zy_3$ so that $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(\$)$, where g(\$) = {A, AA}. Let Z = AA. Then, $y = y_1AAy_3$ and there exists the derivation $S \Rightarrow^*_G y_1AAy_3 \Rightarrow_G y_1y_3$ [AA → ε], where $y_1y_3 = x$, $x \in g(x')$.

(viii) p' = (B → #B', 0, B'). Then, $y' = y'_1By'_3$, $x' = y'_1\#B'y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(B)$. Because g(B) = {B} the only Z is B, so we can express $y = y_1By_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $B \in g(\#B')$

(ix) p' = (#→ ε, #B', 0). By the permitting condition of this production string #B' surely occurs in y'. By Claim 1, no more than one B' can occur in y'. Therefore, y' must be of form $y' = y_1'\#B'y_3'$, where $y_1', y_3' \in (V')^*$ and $B' \notin sub(y_1'y_3')$. Then $x' = y_1'\#y_3'$ and y is of the form $y = y_1Zy_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(\#)$. Because g(#) = {B, BBB}, the only Z is BBB; thus, we obtain $y = y_1BBBy_3$. By the induction hypothesis, we have a derivation $S \Rightarrow_G y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $BBB \in g(\#)$.

(x) p' = (B' → ε, 0, #). Then, $y' = y'_1B'y'_3$ and $x' = y'_1y'_3$, where $y'_1, y'_3 \in (V')^*$. Express $y = y_1Zy_3$ so that $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(B')$. Let Z = BBB. Then, $y = y_1BBBy_3$ and there exists the derivation $S \Rightarrow^*_G y_1BBBy_3 \Rightarrow_G y_1y_3$ [BBB → ε], where $y_1y_3 = x$, $x \in g(x')$.

(xi) p' = (B → #B', 0, B'). Then, $y' = y'_1By'_3$, $x' = y'_1\#B'y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(B)$. Because g(B) = {B} the only Z is B, so we can express $y = y_1By_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because $B \in g(\#B')$

(xii) p' = (#→ ε, #B', 0). By the permitting condition of this production string #B' surely occurs in y'. By Claim 1, no more than one B' can occur in y'. Therefore, y' must be of form $y' = y_1'\#B'y_3'$, where $y_1', y_3' \in (V')^*$ and $B' \notin sub(y_1'y_3')$. Then $x' = y_1'\#y_3'$ and y is of the form $y = y_1Zy_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(\#)$. Because g(#) = {B, BBB}, the only Z is BBB; thus, we obtain $y = y_1BBBy_3$. By the induction hypothesis, we have a derivation $S \Rightarrow_G y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $BBB \in g(\#)$.

(xiii) p' = (#→ ε, #B', 0). By the permitting condition of this production string #B' surely occurs in y'. By Claim 1, no more than one B' can occur in y'. Therefore, y' must be of form $y' = y_1'\#B'y_3'$, where $y_1', y_3' \in (V')^*$ and $B' \notin sub(y_1'y_3')$. Then $x' = y_1'\#y_3'$ and y is of the form $y = y_1Zy_3$, where $y_1 \in g(y_1')$, $y_3 \in g(y_3')$ and $Z \in g(\#)$. Because g(#) = {B, BBB}, the only Z is BBB; thus, we obtain $y = y_1BBBy_3$. By the induction hypothesis, we have a derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $BBB \in g(\#)$.

(xiv) p' = (B → #B', 0, B'). Then, $y' = y'_1By'_3$, $x' = y'_1\#B'y'_3$, $y'_1, y'_3 \in (V')^*$ and $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(B)$. Because g(B) = {B} the only Z is B, so we can express $y = y_1By_3$. Having the derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$, it is easy to see that also $y \in g(x')$ because B ∈

g(#B')

(xv) p' = (# $\rightarrow \varepsilon$, #B', 0). By the permitting condition of this production string #B' surely occurs in y'. By Claim 1, no more than one B' can occur in y'. Therefore, y' must be of form $y' = y'_1\#B'y'_3$, where $y'_1, y'_3 \in (V')^*$ and $B' \notin sub(y'_1y'_3)$. Then $x' = y'_1\#y'_3$ and y is of the form $y = y_1Zy_3$, where $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(\#)$. Because g(#) = {B, BBB}, the only Z is BBB; thus, we obtain $y = y_1BBBy_3$. By the induction hypothesis, we have a derivation $S \Rightarrow^*_G y$ such that $y \in g(y')$. According to definition of g, $y \in g(x')$ as well because $BBB \in g(\#)$.

(xvi) p' = (B' $\rightarrow \varepsilon$, 0, #). Then, $y' = y'_1B'y'_3$ and $x' = y'_1y'_3$, where $y'_1, y'_3 \in (V')*$. Express $y = y_1Zy_3$ so that $y_1 \in g(y'_1)$, $y_3 \in g(y'_3)$ and $Z \in g(B')$. Let Z = BBB. Then, $y = y_1BBBy_3$ and there exists the derivation $S \Rightarrow^*_G y_1BBBy_3 \Rightarrow_G y_1y_3$ [BBB $\rightarrow \varepsilon$], where $y_1y_3 = x$, $x \in g(x')$.

We have completed the proof and established Claim 2 by the principle of induction. Observe that L(G) = L(G') follows from Claim 2. Indeed, according to the definition of g we have g(a) = {a} for all $a \in T$. Thus, from Claim 2, we have for any $x \in T^*$:

$S \Rightarrow^*_G x$ if and only if $S \Rightarrow^*_{G'} x$.

Consequently, L(G) = L(G') and the theorem holds.

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